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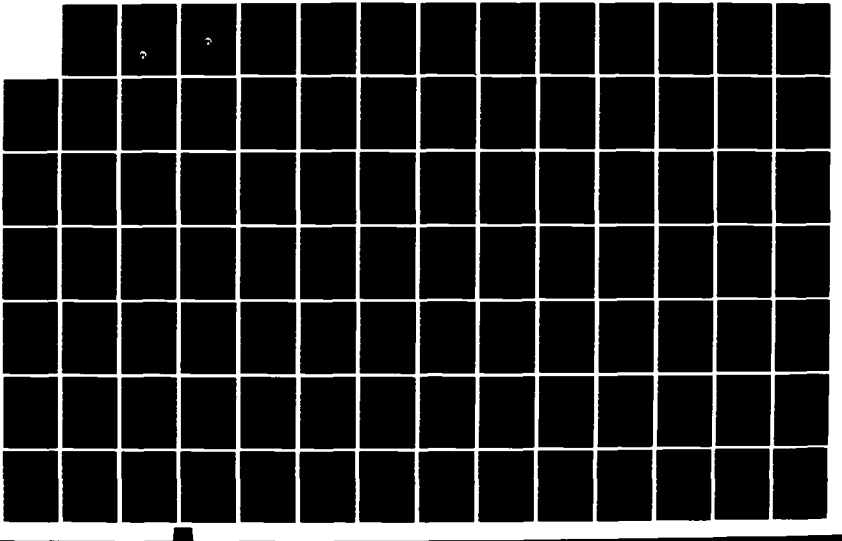

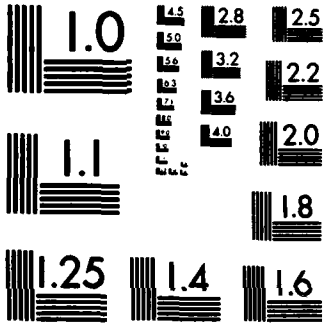

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The Octree Encoding Method for Efficient Solid Modeling

Donald J. R. Meagher

August 1982

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ABSTRACT

Solid modeling is the subject of representing solid objects in a computer - to permit their analysis, manipulation and display. This thesis describes the development of a new solid modeling method called octree encoding, in which arbitrary objects are represented to a specified resolution in 8-ary hierarchical trees or octrees. The number of nodes in an object's octree is used as a measure of object complexity. This number is shown to be on the order of the product of object surface area and the inverse of the square of the resolution.

A dual data-base approach is proposed. A general-purpose solid-modeling system based on octree encoding would interactively perform geometric, analytical and display operations in conjunction with specialized application data bases.

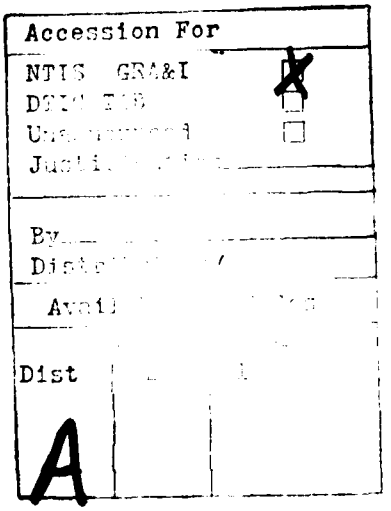
Efficient algorithms are presented for the determination of mass properties (volume, surface area, center of mass and moment of inertia, etc.) for the formation of new objects via the use of set operations (union, intersection, difference and negation), for linear transformations (including translation, scaling and rotation), for interference detection, for swept-volume definition, and for display from any point in space (with

surface texture, anti-aliasing and hidden surface removal). The complexity of the processing required to display an object is related to the visual complexity of the scene rather than the complexity of all objects involved. Interference detection requires computation related to the separation distance between the objects.

The above algorithms require only simple integer arithmetic (addition, subtraction, magnitude comparison, and shift) in order to facilitate implementation in VLSI processors.

The new method is compared to existing solid modeling methods in 21 problem areas.

Results are presented which show the application of the technique in the verification of NC (Numerical Control) machine programming and in the display of 3-D medical objects derived from multiple CT (Computed Tomography) images.



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ACKNOWLEDGEMENT

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CHAPTER 1

INTRODUCTION AND HISTORICAL REVIEW

Solid modeling is concerned with methods and systems for the computer representation, manipulation, analysis and display of solid objects. The primary applications for solid modeling are in CAD/CAM (Computer-Aided Design and Computer-Aided Manufacturing). This includes the design and analysis of mechanical parts, the generation and verification of commands for NC (Numerical Control) machines, the analysis of kinematic chains, and the analysis of space utilization processes (e.g., packaging, process planning, robotics, parts assembly, etc.). Other application areas include medical imaging and cinematography.

Many experts predict that solid modeling will be the key to the future of CAD/CAM [13, 35, 47]. The Solid Modeling Systems (SMSs) being developed today will form the lower levels of advanced CAD/CAM systems. The higher levels will incorporate sophisticated artifical intelligence techniques. They will draw upon vast knowledge structures containing the information needed to automate tasks such as part design and process planning. As such, these lower-level facilities will be utilized with a high duty cycle and, just as with lower-level operating system

procedures, must be made fast and efficient. They must be extremely reliable and completely automatic. Human intervention to resolve ambiguities obviously could not be tolerated.

Although there are currently more than 20 major SMSs, they are all limited to a greater or lesser degree because of deficiencies in object representation and processing algorithms [54]. Two major shortcomings will be mentioned. First, representation capabilities are not sufficiently robust easily to handle the object complexities required in a realistic environment. Second, the manipulation and display algorithms for such functions as interference detection (two or more objects occupying the same space) and hidden-surface removal (necessary for realistic display) require extremely large numbers of calculations in practical situations. They usually exhibit polynomial growth (often quadratic) in the number and complexity of the objects.

The goal of this research has been to devise a new object representation scheme and associated linear growth algorithms in which objects of abitrary complexity can be encoded, manipulated, analyzed and displayed interactively in low-cost hardware. A solid modeling method called octree encoding was developed. The technique is based on a hierarchical 8-ary tree or "octree."

This research was conducted over a period of almost five years. This thesis is the seventh in a series of

publications documenting the work [38-43]. The major results are presented here, with the earlier reports referenced for supporting information.

1.1 Existing Solid Modeling Schemes

Most commercially available CAD systems do not employ a true SMS in that 3-D objects are not really modeled. They are essentially extensions of drafting techniques based on the use of edges to represent solids in projection. The determination of what is actually solid is left to human interpretation. Most systems cannot reliably remove hidden lines or generate sectional views automatically.

Excluding such "drafting" schemes, most existing SMSs employ one or more of the following six representation schemes [55,56]:

(1) Primitive Instancing - families of objects are defined parametrically. A shape type and a limited set of parameter values specify an object.

(2) Spatial Enumeration - an object is represented by the cubical spatial cells (volume elements or "voxels") which it occupies.

(3) Cell Decomposition - a generalized form of spatial enumeration in which the disjoint cells are not necessarily cubical or even identical.

(4) Constructive Solid Geometry (CSG) - objects are

represented as collections of primitive solids (cuboids, cylinders, etc.). A tree structure is typically used with leaf nodes representing primitives and branch nodes specifying set operations.

(5) Sweep Representation - a solid is defined as the volume swept by a 2-D or 3-D shape as it is translated along a curve.

(6) Boundary Representation (B-Rep) - objects are represented by their enclosing surfaces (planes, quadric surfaces, patches, etc.).

Specific advantages and disadvantages of each have been tabulated [57] along with a classification of 21 existing systems. For a primary representation scheme, most use CSG (TIPS, PADL, SynthaVision, etc.) or B-Rep (Build, CADD, Design, Solidesign, Romulus, etc.) or a combination of the two (EUKLID, GMSolid). Often other formats are used for secondary functions. Some allow alternate representations, such as swept volume, for input. Some have special features. For example, TIPS sorts the primitives into a spatial enumeration array to facilitate interference analysis. In a separate study, Baer, Eastman and Henrion [3] have analyzed and compared 11 popular systems. Much information has recently become available on specific systems [7-9, 10, 11, 28, 36, 49, 61, 80, 82].

New methods are needed to solve or at least reduce the

following problems which have been found to plague (to a greater or lesser extent) systems based on the above six schemes (see [27, 56]).

(1) Limited domain - The currently used schemes are characterized by a restricted domain of representable objects because they are constructed from a limited number of mathematically well-defined surface or solid primitives. Some systems allow quadric surfaces and higher-order patches. This can limit performance, however, because the more general and more powerful primitives usually require substantial additional computations for object manipulation and display. Adding a new primitive to a system or generalizing the use of an existing one may necessitate extensive development of mathematical tools and significant software modification. Another consideration is the potentially large labor cost involved in what is essentially the art of fitting primitives to a desired object.

(2) Validity - In many schemes, not all objects which can be created are true or valid 3-D objects (also called "well-formed" objects). The intersection of two objects, for example, may create "nonsense objects" such as dangling edges or faces bounding no volume. Some systems depend upon human intervention to eliminate such artifacts. Others contain ad hoc algorithms to detect and eliminate invalid elements after each operation. A few make such checking an integral part of the manipulation algorithms. In any event, it adds overhead

and reduces the efficiency of a scheme. Ideally, any object which can be legally generated should correspond to a valid object.

(3) Completeness - A complete representation contains all the information required to determine the interior and exterior of an object. There should be no ambiguity. All of the aforementioned schemes are (or can be made) complete representations and, in fact, completeness is required for a true SMS. It may be costly in overhead processing, however.

(4) Uniqueness - If a SMS is unique, there is only one possible symbol structure for a given object, regardless of position or orientation. This simplifies object matching and identification, which may be important in some applications. However, most existing schemes are not unique.

There are two common causes of nonuniqueness: permutational nonuniqueness in which substructures in the symbol structures can be permuted, and positional nonuniqueness in which different representations correspond to the same object but at different positions or orientations.

(5) Conciseness - There are two parts to conciseness. Basically, it refers to the amount of data (bytes, for example) required to represent an object. If an object can be represented in a scheme with fewer words of storage, then it is more concise. A deeper meaning takes into account the size of the domain of representable objects. A valid

comparison of conciseness can only be made between two systems when both are configured to represent the same set of possible objects to the same precision, usually a very difficult, if not impossible, task.

(6) Closure - A SMS has the property of closure if the results of any object operation can be used as input for further operations.

(7) Finiteness - It should not be possible to create objects with infinite volumes.

(8) Null object - It should be easy to determine whether an object is null (contains no volume).

(9) Transportability - Object representations should be able to be transported to alternate computer facilities without inadvertant modifications resulting from a change in word size, floating-point precision, etc. In addition, identical operations on all machines should generate identical results.

(10) Extensibility - The initial implementation of a solid modeling system should be easily extended for modeling larger, smaller or more complex objects, a larger number of objects, sculptured surfaces, objects to a greater precision, etc.

(11) Autonomy - The scheme should not require human intervention under any circumstances.

(12) Reliability - All possible objects (or at least all objects which could be required) should be able to be

processed by all operators without error. In some systems, for example, self-intersection is a fatal condition.

(13) Efficiency - The computational complexity and resource requirements (memory, for example) of algorithms should not grow faster than linearly in the number and complexity of objects. In addition, it would be very desirable if the instantenous computational load were related to the actual complexity (in some sense) of the specific user requested task rather than the overall number and complexity of objects involved.

(14) Implementability - An SMS should be easily implemented for a large range of practical tasks on existing computers or in hardware utilizing current or near-term technology. A scheme will be more easily implementable if the number of calculations are small relative to the number of objects and object complexity. Also, algorithms requiring simple mathematical operations and those allowing extensive paralleling or pipelining of operations will be easier to implement in the VLSI (Very-Large-Scale Integration) environment of the future.

Many of the existing SMSs were designed when an evaluation of the hardware requirements called for compact data structures and algorithms that would fit into limited memory (typically 64K bytes). Calculations were to be handled by a single, serial processor, most often a general-purpose minicomputer. This stategy has resulted in

schemes that may be very efficient in memory usage but often require unrealistically large numbers of calculations, usually in floating-point rather than integer arithmetic. A more realistic appraisal of current hardware trends could result in more powerful and easily implementable schemes.

(15) Multiple representations - For a broad-based application, two or more representation schemes may need to be maintained to handle all requirements. Experts in the field seem to agree (reluctantly) that this will be necessary, given the limitations of the six representation schemes.

(16) Consistency - If multiple representation schemes are maintained, they should be guaranteed to be consistent at all times. If two representations of a single object could be contradictory, an application system would probably be useless.

(17) Conversion - If multiple representations are used, a method to convert from one to another is required. The transformation should be exact (rather than approximate) and invertable. In general, however, this is not the case with existing schemes.

(18) Ease of object creation and manipulation - In general, a system that allows a user more easily to generate and manipulate any desired object will be more useful. A short response time is desirable, for example.

(19) Finite-element modeling capability - It is an

advantage if a scheme can easily and automatically generate a finite-element mesh in 3-D.

(20) Interference analysis - Many applications require the detection of interference between two or more objects. The generation of a hidden surface view can be thought of as a visual interference problem. The development of linear growth algorithms involving interference was a major objective of the research described in this thesis.

(21) Tweaking - It should be possible to make local changes without involving the entire object.

1.2 Historical Review

The solid modeling method described here results from a doctoral program undertaken by the author in September 1977. In the spring of 1978 a literature survey of techniques for representing and analyzing shapes was carried out, with an emphasis on 3-D techniques [38].

In the fall of 1978 the general goals of the research effort began to take form. A new solid modeling method would be developed to alleviate many of the common problems encountered with existing systems, especially when applied to large-scale efforts in CAD/CAM. The primary consideration was to develop a simple, powerful, efficient and fast scheme that could be easily implemented in VLSI for future "real-world" applications of great complexity.

The coding of what later became the program OCTREE began in October 1978. The outline of a new technique employing hierarchical tree structures was defined and first implemented in a 2-D scheme called "area encoding." By the spring of 1979, algorithms had been developed and implemented for the entry of 2-D chain codes, conversion to an area encoded format, and generation of the intersection of two objects.

At about the same time, April 1979, the first widely distributed paper describing a similar 2-D technique with applications in a different area, image processing, was published by Hunter and Steiglitz [29]. Based on Hunter's PhD thesis [30], it made use of 2-D hierarchical tree structures called "quadtrees" (also called "quad-trees" or "quad trees").

Further investigation turned up a proposal by Tanimoto [78] to use a "pyramid" image model as a measure of binary image complexity. Additional quadtree publications have appeared in the literature since that time. Rosenfeld [58, 59] has pursued quadtree efforts in pattern recognition and image processing. Samet has presented an overview of quadtrees (with Rosenfeld) [72], and developed quadtree algorithms to compute the perimeter [65], convert from boundary codes [68], to boundary codes (with Dyer and Rosenfeld) [19], from raster format [69], to raster format [70], from binary arrays [71], compute the medial axis

transformation [63], compute a distance function [64], find neighbors [67], and label connected components [66]. Shneier has developed algorithms to calculate geometric properties of quadtrees [74]. Ranade has employed quadtree techniques for edge enhancement [51], and (with Shneier) for image smoothing [50, 52]. Hunter and Steiglitz have presented an algorithm to perform linear operations on quadtrees based on transforming edge segments [31]. Burt has developed the "hierarchical discrete correlation" (HDC) for efficient image processing [12].

The effort continued during the summer of 1979 with the development of the "overlay" technique for efficiently performing linear operations on objects and the basic hidden surface display algorithms. Much of this was implemented and verified during the fall of 1979.

In October 1979 a presentation of the technique was made that included computer output examples of translation, rotation (90-degree, 180-degree and arbitrary-angle), union, intersection, difference, reflection about an axis, 2-D hidden-line elimination and 2-D perspective display. At about this time the name "octree encoding" was given to the technique (it had earlier been called "volume encoding").

In the spring of 1980, a report was written to present the scheme. It was first submitted in July of 1980 and later published as an IPL technical report [39]. Additional results were presented in a paper titled "Geometric Modeling

Using Octree Encoding." It was submitted in December 1980 and released as an IPL technical report [40]. A slightly updated version was published in Computer Graphics and Image Processing [44].

The Octree Encoding scheme was presented in the Computer Graphics course at RPI during the Fall of 1980 and officially proposed as a PhD thesis topic in March 1981.

The advanced display algorithm presented below was developed during the spring and summer of 1981. It was first described early in September 1981 and, later that month, a paper describing the technique was presented at the IEEE Computer Society's Tenth Workshop on Applied Imagery Pattern Recognition (AIPR) in College Park, Maryland. The paper was also released as an IPL technical report [41]. An updated version was presented at the IEEE Computer Society's Pattern Recognition and Image Processing (PRIP) conference in June 1982 [45].

The "hierarchical convolution" technique and its application to swept volume generation were developed during the Fall of 1981. The swept volume algorithm was first presented in November 1981. The algorithm is described below.

A major effort in late 1981 and early 1982 was devoted to efficient object generation algorithms. These and other results are documented in an IPL technical report [43].

The general idea of hierarchical geometric models as the

basis for future hidden-surface algorithms was proposed by Clark [16]. Multidimensional binary trees and algorithms have been studied by Bentley for use in data base applications [4, 6]. Franklin has developed the "variable grid" technique for hidden line and surface applications [21-23]. It is shown to be a linear growth algorithm at the expense of pre-sorting. This has been extended into a hierarchical structure in Octree Encoding and forms the basis of the linear computational characteristics of the scheme.

Rubin and Whitted [62], Reddy and Rubin [53], Fuchs [25], and others have presented various object space pre-sorting techniques. The use of 8-ary hierarchical trees to represent 3-D objects was apparently first suggested by Hunter [30] in his PhD thesis (1978) as a possible extension of quadtrees. It was later independently proposed by Jackins and Tanimoto [33, 34], Moravec [46], Srihari [75, 76] for medical imaging, Meagher [39] and perhaps others. Later octree reports include [17, 18, 32, 86].

1.3 Approach

The ultimate goal of this effort has been the actual construction of a full-function, real-time (about 1/30 second response) solid modeling system to handle any number of arbitrarily complex objects while operating on relatively low-cost hardware. Conventional solid modeling wisdom

assumed that these characteristics were mutually exclusive. Levels of performance many orders of magnitude greater than allowed by existing techniques are needed. Obviously a new approach was required!

The remainder of this section is a summary of the four or five years of evolving reasoning and philosophy embodied in the octree encoding method. A more in-depth study is presented in [43].

The first step was to reject any preconceived ideas about solid modeling. An entirely new method would be designed and developed. A list of priorities was established for guidance and direction. The highest priority throughout the effort was high-speed operation. For the first few years the actual usefulness of the method was open to question but there was never doubt the functions could be performed at an extremely high throughput utilizing modest hardware.

The second priority was robustness. This included both the ability to represent arbitrary objects and a full complement of analysis, manipulation and display functions.

The third thrust was a general drive for simplicity. This required, for example, a single representation scheme for all objects and very simple algorithms.

Given these general priorities, the first step was to devise a solution to the object storage problem. Arbitrary objects require arbitrary quantities of storage for representation. It was decided to represent arbitrary

objects to a variable but limited precision.

CSG schemes handle this dilemma by taking what can be looked at as the opposite choice. Unlimited resolution (for all practical purposes) is preserved but objects are restricted to primitive analytic shapes or combinations thereof.

The next step was to address the computational complexity issue. Most existing CAD systems have evolved from attempts to automate 2-D drafting. Complexity has not generally been a consideration because the typical drafting task is linear in a small number of items. Interactive operation is not difficult to achieve.

The progress of CAD into full 3-D applications has changed the situation. Operations involving some form of interference analysis have been found to require large, often prohibitively large, computational resources for interactive operation. The root of the problem is a comparison task. Naive algorithms perform an interference detection operation by checking for intersection between each possibly relevant pair of primitives. A combinatorial explosion results because, in general, the number of pairs grows quadratically in the number of primitives.

The solution was to design a spatially pre-sorted representation scheme that would never require additional sorting or extensive searching. The octree scheme satisfies this requirement.

The philosophical approach adopted for algorithm development was based on the hierarchical ideas of Clark [16] and the sharing of partial calculations that has been proved so successful in the Fast-Fourier Transform (FFT). It is the hierarchical structure into which the problem has been cast that allows large numbers of low-level calculations to be eliminated at an early stage when processing typical objects.

The approach to actual implementation adopted for this effort was based on the current trends in VLSI technology. It is clear that to maximize the performance-to-cost ratio, full advantage should be taken of the tremendous improvements in hardware which have resulted and will no doubt continue to result from VLSI.

Before proceeding, a popular misconception concerning increased computing power should be dispelled. The thought that increasingly powerful hardware at lower cost will allow inefficient algorithms to become useful is, in general, wrong. Other factors being equal, a performance increase will allow larger problems to be handled, further widening the performance gap between an inefficient (quadratic growth, for example) algorithm and an efficient (linear) algorithm. Thus, computational complexity issues become more, not less, important as technology moves into the VLSI age.

The implementation approach was to develop algorithms designed specifically for semi-custom or full-custom VLSI based operation. This decision impacted the entire design

philosophy. Traditional solid modeling systems tend to be huge, ever growing, ever changing, software packages with complex internal structures. The VLSI based system, in contrast, must be based on a small number of very simple, fixed, powerful and extremely reliable algorithms. They are implemented in hardware and form the primitive lower level functions in an applied system.

Based on the simplicity and ease of implementation requirements, algorithms were allowed to employ only integer numbers and only simple arithmetic (addition, subtraction, magnitude comparison and shifts). Neither floating-point operations, integer multiplications nor integer divisions were allowed.

CHAPTER 2

THE OCTREE METHOD

2.1 Definitions

A graph G(N,P) is a finite, nonempty collection N of nodes and a set P of unordered pairs of distinct nodes called edges. Two nodes connected by an edge are adjacent nodes. If an edge has an associated direction, it is a directed edge. The direction is from the tail node to the head node. A graph containing only directed edges is a directed graph. The number of edges having a particular node as their tail node is the outdegree of that node. The number of edges having a particular node as their head node is the indegree of that node.

A path is a sequence of edges connecting two nodes. For a directed graph, the nodes visited must be in tail-to-head order. A graph containing no paths which originate and end in the same node is called acyclic.

A tree is an acyclic directed graph in which all nodes have indegree 1 except one node, the root, which has indegree 0. Any node with outdegree 0 is called a terminal node or leaf. Nodes with outdegree greater than 0 are branch nodes. The level of a node is defined as the distance in edges from

the root. The root is at level 0.

The root is assumed to be at the top of the node structure and all other nodes exist below the root. All nodes reachable from a particular node are called the descendants of that node. All nodes from which a particular node can be reached are the ancestors of that node. Descendants one level below a node are the children of that node. The ancestor adjacent to a node is the parent of that node. Nodes having a common parent are siblings.

If a node does not actually exist in a tree but can be inferred from an existing terminal node which would be one of its ancestors, it is called an implied node. Loosely, operations on a tree which use implied nodes are said to process the implied tree rather than the actual tree.

Every branch node is a root of one or more subtrees. The degree of a node is the number of subtrees that exist for that node. If the outdegree of every branch node is <= m, the tree is an m-ary tree. If the outdegree of every branch node is m, the tree is a complete m-ary tree.

An m-ary tree is positional if the children have m distinct positions. The position is indicated by a value from the child number set {0,1,2,...,m-1}. Every node is uniquely identified by a string over the child number set, the node address. The root is represented by the empty string. The node address of a child is the child number prefixed by the address string of its parent.

A tree will be called a hierarchical tree if the children of a node are associated with their parent in some particular relationship.

All objects exist within the universe. It is a finite section of N-dimensional space defined by N orthogonal axes and $0<=x(i)<=d$ where $x(i)$ is a displacement in dimension i, $(x(1), x(2),...,x(N))$ is a point in the universe, d is the length of an edge of the universe and N is the order of the universe (number of dimensions). The symbol "N" will be reserved for the order of the universe throughout this thesis.

Note that all edges of the universe have the same length and form a square for N=2, a cube for N=3 and an N-dimensional hypercube for N>3. The origin of the universe is the point of intersection of the axes. Negative displacements from the origin are not allowed. The space beyond the universe is the void. No object can exist in the void. Any part of an object moved into the void is annihilated. An augmented universe is one in which one or more adjacent (empty) universes are added to the primary universe. Augmented universes are used to facilitate algorithm initialization.

Before encoding, objects are called real objects. They may be real-world objects or a mathematical description of an ideal shape. An object encoded in the octree format is known as the encoded object or simply the object.

If a single encoded object is used many times to generate new, transformed objects, the original object is the model and the new objects are instances.

An object is always of the same order as the universe in which it is defined and is composed of discrete units of N-dimensional space. All objects in a third order universe must occupy volume, for example. A 2-D object could not exist here. The smallest object in such a universe would be the smallest resolvable unit of space.

Other than this, there are almost no restrictions on objects. They can be concave as well as convex, have interior voids, and can be simply- or multiply-connected.

Each object is defined over the entire universe. It has a property value defined at each point in the universe. For a typical small object (relative to the universe) most of the space in the universe has the property of being empty.

2.2 Object Representation

During the design and development of the object representation scheme, the primary considerations were the need for a spatially sorted format to eliminate the quadratic growth of algorithms and the desirability of a hierarchical structure to reduce the volume of data that would need processing for a typical solid modeling operation. A third consideration was the simplicity of algorithms that could

result if a regular spatial structure was used.

To facilitate spatial sorting, orthogonal planes were used to segment object space. The need for a hierarchical structure was satisfied by employing trees in which the children represented the same space as the parent but to a higher precision. To fulfill the requirement for a regular structure, nodes at a given level represent disjoint segments of space, are identical size, shape and orientation, and completely fill the universe when all possible nodes are taken together. The result was a recursive subdivision of space with objects represented by cubes of exponentially related size.

An octree object is represented by an 8-ary hierarchical tree or octree. Each node represents a cubical section of the universe and contains a property value associated with the cube. In its simplest form, the property has one of 3 values. If the space is completely occupied by the object, the node has the value F (for "full"). If completely disjoint, the value is E (for "empty"). If neither occupied nor disjoint (at least part of the object's surface is within the cube) it has the value P (for "partially occupied"). The property value is often loosely used as a node qualifier. For example, a "P node" is a node with a property value of P.

The 8 octants of the cube represented by a node are, in turn, represented by the node's 8 children. An octree is hierarchical in that the children, taken together, represent

exactaly the same space as the parent.

The correspondence between octant and child number is defined in 2.1 along with the vertex labeling convention. A sample object and the universe are shown in 2.2(a). In 2.2(b) the corresponding octree is presented. The root node at level 0 represents the entire universe and is given the value P. At level 1, of the 8 octants of the universe, 6 are empty and given E values. Two are partially occupied and are given P values. At level 2, the three solid cubes forming the object result in three F nodes. The node addresses are shown below the nodes.

E and F nodes represent homogeneous sections of space. There is no need for further subdivision and they are, therefore, leaf nodes. The space represented by a P node is not homogeneous. Lower level nodes are needed to resolve the object. P nodes are thus branch nodes.

This scheme can be applied over any number of dimensions. A 1-D hierarchical binary tree is a bitree. In 2-D, it is a quadtree, in 3-D, an octree and in 4-D, a hexadecatree.

The segments of space represented by nodes are object elements or obels. They are object elements even if disjoint from the object (E nodes). Obels are distinguished from spatial enumeration voxels because they are not uniform in size and not necessarily three-dimensional. An obel is a section of N-dimensional space whereas the node representing

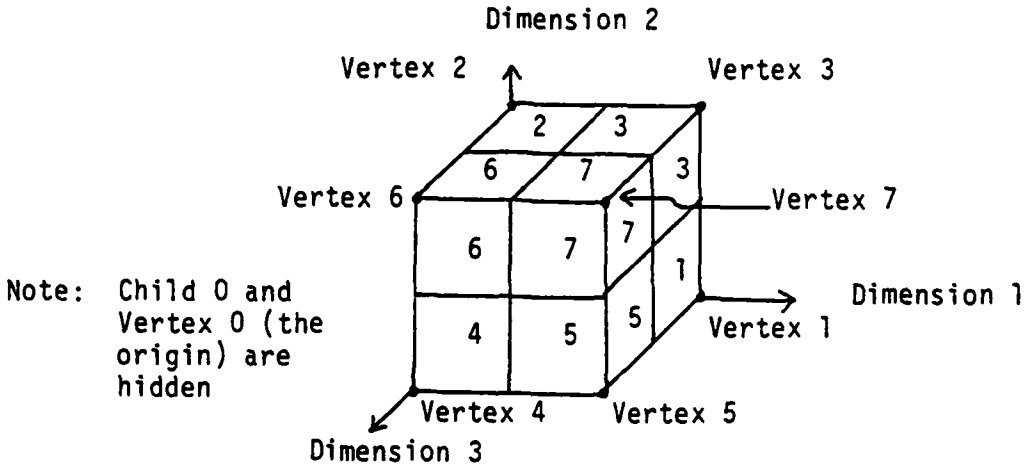
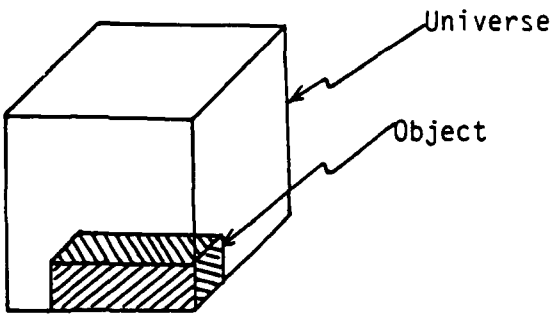
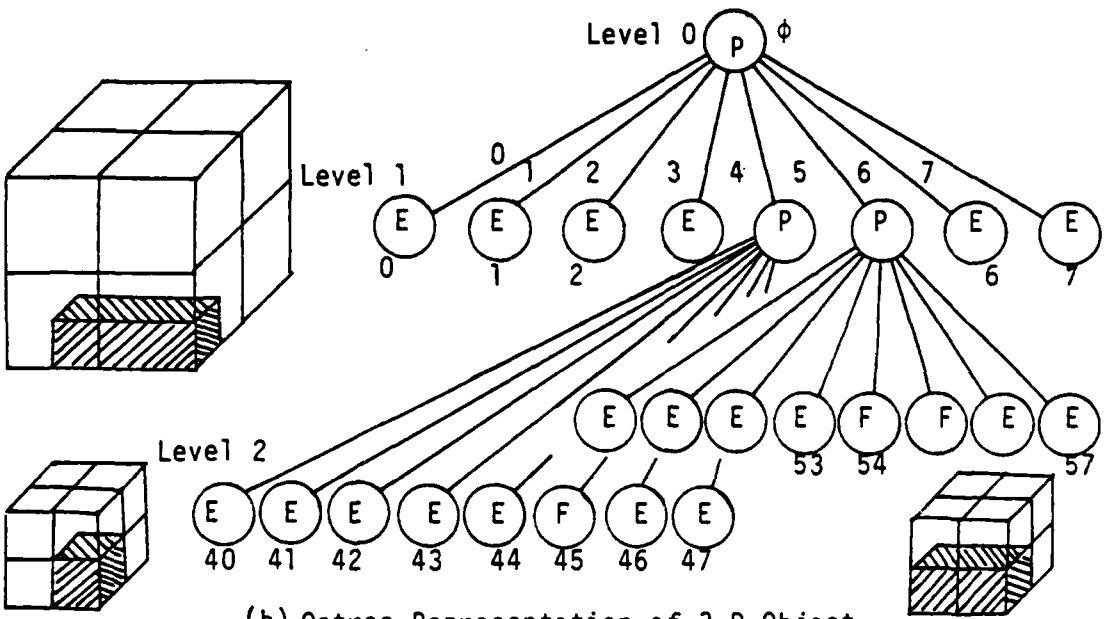


Figure 2.1 Child and Vertex Labeling



(a) 3-D Object



(b) Octree Representation of 3-D Object

Figure 2.2 Sample Octree

it is an element of a data structure but they are generally considered to be synonymous.

Trees as used here should not be confused with the non-hierarchical tree structures used in data processing to maintain items in a sorted format. Adding to the confusion is the fact that such structures have also been called "quad trees" by Finkel and Bentley [20] when representing data items with a double index.

In advanced applications additional properties such as color or texture values, material type, function, density, surface normals, thermal conductivity, etc. are simply attached to F nodes and possibly P nodes.

When a real object is converted to octree format, branch nodes at the lowest level must be given a terminal value. This could be E or F if the obel is less than or greater than half occupied, respectively. If interference detection is involved, the worst case situation is usually assumed, in which case they are given the value F.

If all of the children of a branch node are terminal with the same property value, the children are unnecessary. They should be eliminated and the parent converted to a leaf node containing the child property. If a tree contains no such nodes it is a <u>reduced</u> or <u>trimmed</u> tree.

Unreduced trees are sometimes created during algorithm operation. They are legal objects and are correctly processed by most algorithms but cause inefficiency.

Versions of algorithms that generate trees in a depth-first sequence typically eliminate unnecessary nodes during operation. The output is a reduced tree. Otherwise a separate reduction pass through the tree is usually performed.

In specific applications an additional terminal node type, a tolerance node, is used. In a material removal operation they represent the tolerance object. This is the space within the tolerance of the surface of the minimum desired object which can be optionally removed. Tolerance nodes are handled in a special manner by processing algorithms. They can be used as E nodes to obtain the minimum object, as F nodes for the maximum object or locally converted to whichever would result in the greater node reduction for the minimum storage object (assuming tolerance information no longer needed).

It should be noted that the location of any obel in the universe is known exactly. The limited precision of an object as a function of tree level applies to the location of the surface of the object within an obel.

The node address identifies a particular node and also locates the section of space represented by the node. In a 1-D tree it is a binary string. The number of bits is equal to the level of the node. The value is the number of the section of the 1-D universe occupied by the node, numbered from 0 at the origin to 2^n-1 where n is the level. In 2-D,

the address will be a string of single digit quaternary numbers and in 3-D, octal numbers. The section of a higher order universe can be determined by independently considering the individual bit for each dimension in the child number values. On occasion a node is identified by its level and the decimal equivalent of the binary value of its address string.

A point can be determined to be interior or exterior in log time by traversing from the root to a leaf node. The child containing the point is selected at each level. For points on a face, edge or vertex of an obel, two, four or eight leaves may need to be examined, respectively.

2.3 Node Requirements

The number of nodes required to represent an object is a function of the size and shape of the object, its position and orientation when digitized, the level of resolution, etc. An upper limit is set, however, by object-surface area and the resolution of the object. The following is similar to the proof presented by Hunter and Steiglitz [29] showing the number of nodes required for a 2-D quadtree object to be on the order of the perimeter of the object.

Assertion 2.1: For a representable 3-D connected object, the number of nodes required for octree representation is on the order of the product of the surface

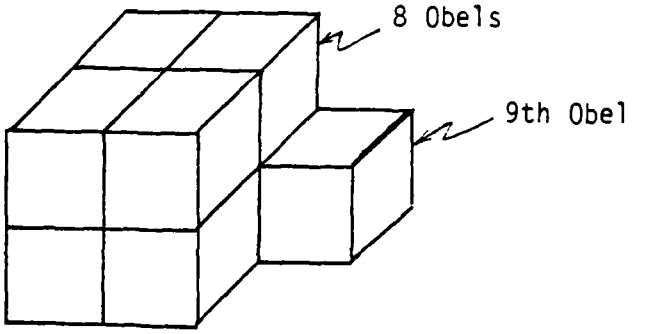
area of the object and the inverse of the square of the resolution.

Proof: A function g(n) is defined to be on the order of f(n), or O(f(n)), if there exists a constant c such that g(n) <= cf(n) for all but some finite (possibly empty) set of non-negative values of n.

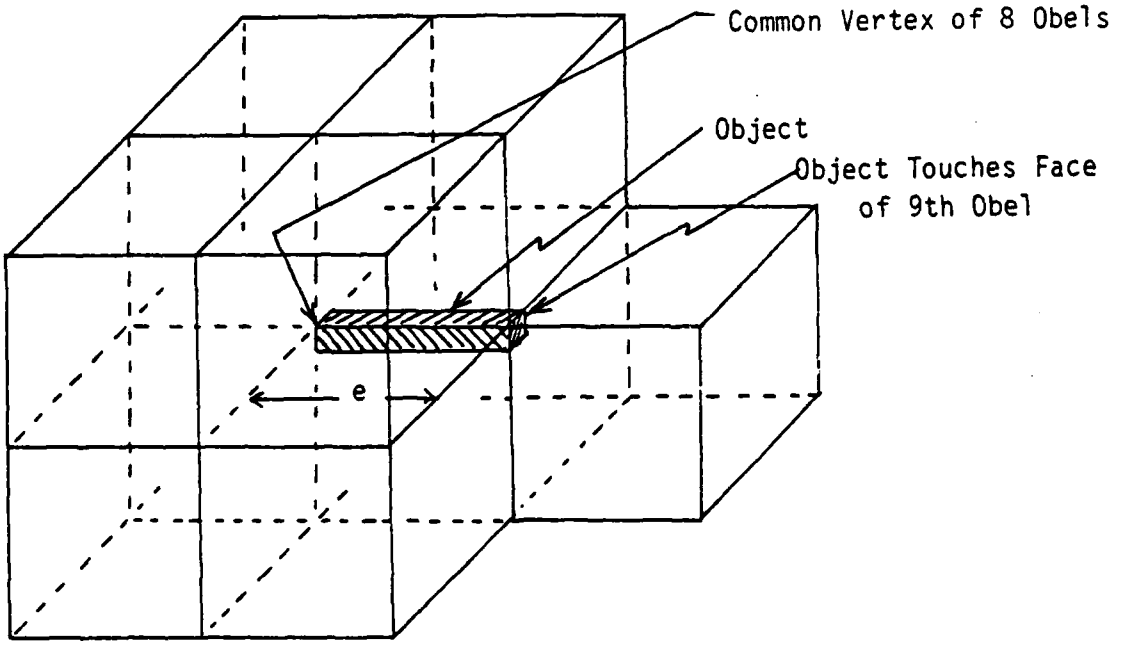
Consider a 3-D universe defined to level m. Without loss of generality, the volume of the universe is defined to be 1. Resolution at level n will be defined as the edge size, e, of an obel at that level or $e=1/2^n$. The resolution of the object, r, is the edge size at the lowest level or $r=1/2^m$.

A representable object is an object that can be represented as a collection of obels. Consider the minimum surface object which touches or intersects 8 obels at level n and continues on tc touch a ninth as shown in Figure 2.3(a). As noted in 2.3(b), it touches or intersects the 8 obels at and around the common vertex at which all 8 touch and continues along the entire length of an edge. It will be a linear run of minimum-level obels for a distance e and has a surface area of $4re+2r^2$ as shown in Figure 2.4. For an object to actually intersect all 9, it must be larger than this and have a larger surface area.

Let S be the surface of an object and let k be the number of cubes at level n which could be enclosed by the object or intersected by its surface. In a worst case



(a) Set of 8 Obels Plus Ninth



(b) Object Touches 8 Obels at Common Vertex and 9th Obel

Figure 2.3 Minimum-Surface Representable Object which Touches 9 Obels

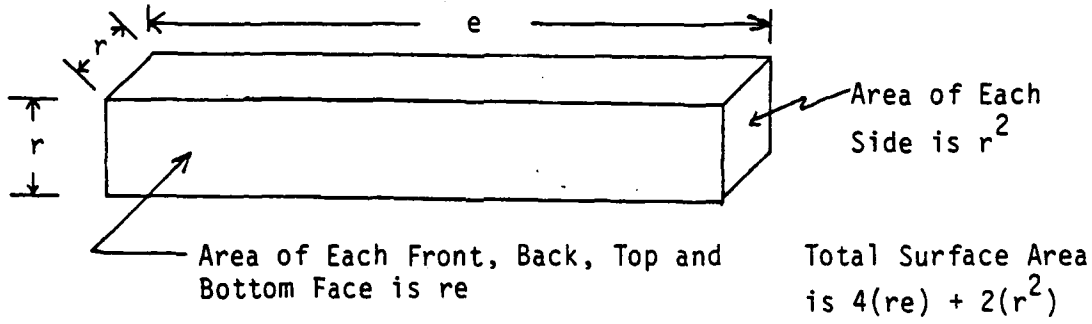


Figure 2.4 Area of Object

situation, the surface area would cover a maximum-length linear run of minimum-level obels. This sets an upper bound on the number of enclosed or intersected obels at level n:

$$k < 8(S/(4re+2r^2)+1)$$

The 1 accounts for (actually, more than accounts for) the four obels which could be intersected along with obel 9 at the far end of the run. Since r^2 is positive, its removal from the denominator will preserve the inequality:

$$k < 8(S/4re+1) = 2S/re+8$$

The value of k is the number of F and P nodes at a level. To place an upper limit on the total number of nodes at level n, it will be assumed that each will have seven E-valued siblings. An upper limit on the node count at a level will thus be 8k.

Let C be the total number of nodes (over all levels) required to represent an object:

$$C < (8)\sum_{n=0}^{m}(2S/re+8)$$

$$C < (16S/r)\sum_{n=0}^{m}(2^n) + 64\left(\sum_{n=0}^{m}(1)\right)$$

$$C < (16S/r)(2^{m+1}-1)+64(m+1) = (32S/r)2^m-16S/r+64m+64$$

$$C < 32Sr^{-2}-16Sr^{-1}+64m+64 \text{ or } C \text{ is } O(Sr^{-2})$$

Q.E.D.

2.4 Complexity Metric

An important item that is generally lacking in the field of 3-D solid modeling is a measure of object complexity. It is difficult to study a situation analytically when quantitative measures are not available. Intuitively, the measure of the complexity of an object should in some sense be related to the amount of information required to represent the object.

The measure of object complexity used here is the number of nodes in its octree [40]. This is an extension of the measure proposed by Tanimoto for binary images [78].

In the remainder of this report the symbol "C" will be reserved to represent the node count (any type) in an object's tree. The value of C is the sum of the number of branch nodes, B, and leaf nodes, L. Because each branch node has 2^N children, and each node (except the root) has a branch node parent, the following relationships hold:

$$C = B + L = B(2^N) + 1 \qquad (2\text{-}1)$$

$$B = (C-1)2^{-N} = \lfloor (2^{-N})C \rfloor \qquad (2\text{-}2)$$

$$L = C-(C-1)2^{-N} = (1-2^{-N})C+2^{-N} = \lceil (1-2^{-N})C \rceil \qquad (2\text{-}3)$$

A tabulation by N is:

	1-D	2-D	3-D	4-D
B =	$\lfloor C/2 \rfloor$	$\lfloor C/4 \rfloor$	$\lfloor C/8 \rfloor$	$\lfloor C/16 \rfloor$
L =	$\lceil C/2 \rceil$	$\lceil 3C/4 \rceil$	$\lceil 7C/8 \rceil$	$\lceil 15C/16 \rceil$

Within the count of leaf nodes, the number of nodes with a value of E (or F) can range from 1 to L-1.

For mathematically simple shapes, the octree C value may be much larger than some object complexity measure which could be defined for a CSG or B-Rep scheme. This disadvantage may be offset by the following: (1) as object complexity increases, the CSG or B-Rep value may approach or exceed (in some sense) the octree C value, and (2) many operations (set operations, for example) exhibit linear growth using octree methods (because of spatial pre-sorting) but quadratic growth using CSG and B-Rep.

Often the number of calculations required within an algorithm is proportional to the C value for an input or output object. Depending on the algorithm and the situation, such a tree (or an intermediate internal tree) may not be reduced. The C value in such a case is the complexity of the tree structure used and may be larger than the reduced C value. On the other hand, because of the hierarchical nature of the octree, many algorithms require only a subset of the input nodes. In such situations processing is proportional to the actual number of nodes accessed.

2.5 Storage Requirements

A minimum usable scheme requires two types of data items per node, a property value and pointers to its children (if a branch node). Additional data items which could be used are parent pointers, multiple property values, average subtree properties, sibling pointers, object feature pointers, pointers into application data structures, etc.

Normally a two-bit field is used to encode the three node values (E, F and P) requiring 2C bits or C/4 bytes for an object. A saving of between about 20% and 44% can be realized by allowing for a single-bit value [43].

Conceptually, each branch node of an octree has 8 storage fields for child pointers. For implementation, however, a single location will suffice because it is a complete tree. The children can be located in blocks of 8. A single pointer to the block will uniquely locate each child. The address of a particular child is simply the sum of the pointer and its child number (0 to 7). In Figure 2.5 a single word per node (4 bytes/word) holds both the value field and pointer field for the object from Figure 2.2. The pointer field for leaf nodes could be used for the storage of additional properties.

Memory requirements can be substantially reduced if node storage is sequentially allocated. Using a heap-like storage format, the position of a value in a string indicates its

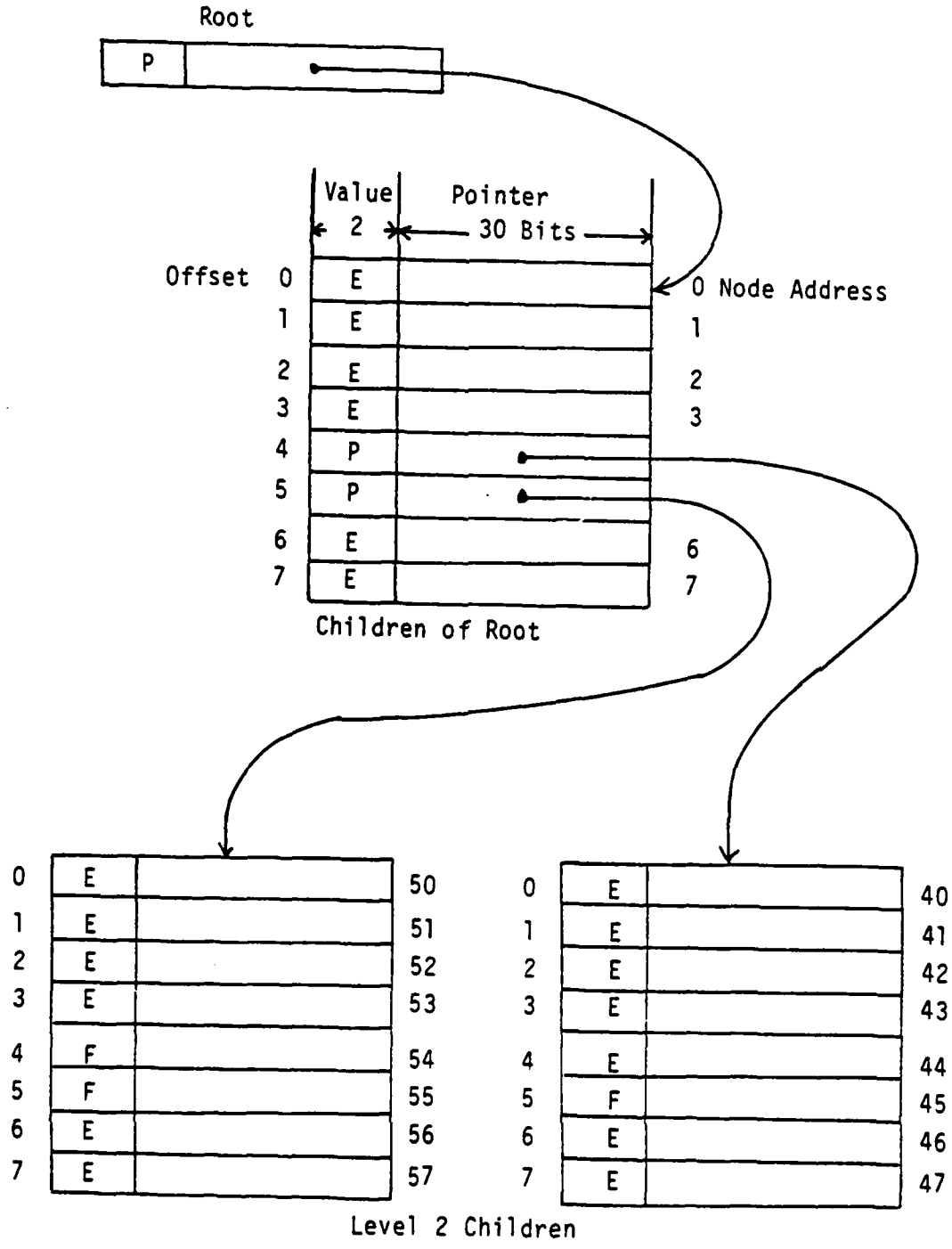


Figure 2.5 Single 4-Byte/Node Storage Format

tree address. Two bits per node (or less) is sufficient. The allocation can be breadth-first or depth-first. The major disadvantage is that, similar to a magnetic tape, all earlier nodes must be read before a desired node can be located.

Parent pointers are probably not necessary because in any real application the number of levels is limited. With a 32-level octree, for example, objects could be represented to a resolution of 0.001 inch in a universe enclosing 311,482.8 cubic miles. In such a situation, depth-first traversal algorithms could keep parent pointers in a small stack.

In some applications subtrees can be shared within an object or between objects. Pointers are simply allowed to point to the same node (root of the shared subtree). This may, however, complicate object modification and deletion.

2.6 Expected Performance

A preliminary analysis of the viability of a real-time solid modeling system based on octree encoding method will be attempted by relating the value of C to the size of the active workspace and by then speculating on the performance of specialized hardware processors.

An important statistic when analyzing the number of nodes required to represent an object is the average ratio of branch children to leaf children. It would be desirable to

calculate an expected value for the number of the children of a node through which the surface of an object would pass, given that the surface passes through the parent node. This number of children will correspond to the number of branch nodes. Nodes not intersected by the surface of the object are completely interior or exterior to the object and will be leaf nodes.

It will first be assumed that the object surfaces cutting the obels are planar. This becomes true even for sculptured surfaces as the obel size becomes very small relative to surface curvature.

A 2-D obel is shown in Figure 2.6(a) along with an edge of the object. Depending on the slope of the intersecting edge, there will be at least one diagonal (segment 1 to 2 in this case) intersecting the edge. Figure 2.6(b) shows the four children and their diagonal lines. Note that the length of a child diagonal is exactly one-half the length of the parent diagonal. Since they are all parallel, if the object edge can intersect the parent diagonal at any random location, it is expected that any particular child will have a 0.5 probability of intersecting the edge. The reasoning is easily extended into 3-D with the same results. Thus, from this analysis, half of the child nodes can be expected to be branch nodes.

In 3-D, an average of four of the eight children of a branch node would thus be expected to be branch nodes. The

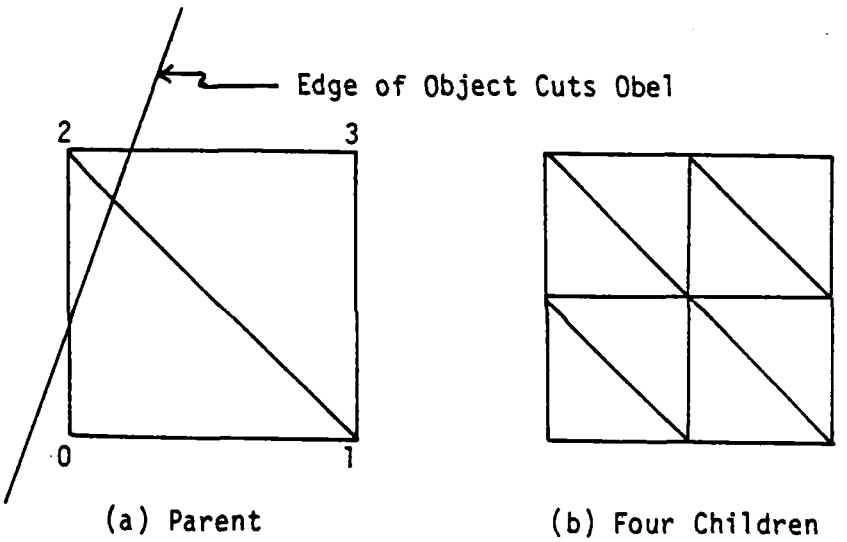


Figure 2.6 Edge of Object Cuts Obel and Intersects Obel Diagonal

number of branch nodes at level i would be 4^i. The total number of nodes at a level must be twice this to account for the leaf nodes except for i=0 which has a single branch node (the root). The expected value of C is thus the summation of the nodes for each level or:

$$C = \sum_{i=0}^{n} (2(4^i))-1 = 2((4^{n+1}-1)/3)-1$$

$$= (2/3)4^{n+1}-5/3 = (2/3)4^{n+1}-1 \qquad (2\text{-}4)$$

where n is the lowest level.

The growth of C by a factor of 4 with each additional level is consistent with the above result showing C to be related to the inverse of the square of the resolution.

It should first be noted that the above rate of node growth is expected only within a section of the universe of size comparable to the size of the object. For a small object within a large universe the node count would be expected to increase by a constant value per level until the obels were of approximately the same size as the object or smaller.

In Table 2.1 a number of items have been tabulated as a function of the number of levels in the universe (assuming the whole universe is the active workspace). The second column is the level number of the lowest level. Column 3 is the resolution of the universe. In a universe with 11 levels, for example, the edge of the smallest obel is 1/1024

Levels	Lowest Level	Resolution (1 part in)	Voxels	Nodes (level N)	Total Nodes	Storage (bytes)
1	0	1	1	1	1	1
2	1	2	8	8	9	3
3	2	4	64	32	41	11
4	3	8	512	128	169	43
5	4	16	4K	512	681	171
6	5	32	32K	2K	2.7K	683
7	6	64	256K	8K	10.6K	2.7K
8	7	128	2M	32K	42.6K	10.7K
9	8	256	16M	128K	171K	42.7K
10	9	512	128M	512K	683K	171K
11	10	1K	1G	2M	2.7M	683K
12	11	2K	8G	8M	10.7M	2.7M
13	12	4K	64G	32M	42.7M	10.7M
14	13	8K	512G	128M	171M	42.7M
15	14	16K	4T	512M	683M	171M
16	15	32K	32T	2G	2.7G	683M

Table 2.1 - Tabulation of Resolution and Expected Node Count

of the edge of the universe. The next column lists the number of voxels (lowest-level obels) in the universe. This is the number of storage locations required in a spatial enumeration representation.

The fifth column lists the expected number of nodes at the lowest level assuming that one-half of the children of branch nodes are also branch nodes.

The sixth column is the accumulated number of nodes (C). It should be emphasized that this value is a rough estimate based on simplistic assumptions. The object must be of comparable size as the universe. In a much larger universe, the C values can be thought of as applying to objects encoded to a resolution relative to the object of approximately the value given in the third column. Also, uniform resolution is not required over the entire surface of an object. Given a constant value of C, lower resolution over some sections will allow higher resolution in other sections.

The last column is the storage requirement for the object, sequentially allocated at 2 bits/node.

Given this information, what level of performance can be expected from a specialized octree processor? It will be assumed that a single unit could process a node in 50 nsec. to 100 nsec., depending on the algorithm and function. If 30 complete operations per second are required for real-time operation, an object with a C value between 325K and 650K nodes could be handled by a single processor. According to

the table, this corresponds to an object defined over a section of the universe of a size between 256^3 and 512^3. Based on this admittedly crude analysis, for algorithms which could be operated in parallel such as display, an 8-processor system would seem to be able easily to handle objects with an average resolution of 1 part in 1024 in each dimension at real-time rates. This assumes a single object in a worst-case situation (all nodes accessed). Such a system would seem to be sufficiently powerful for most interactive situations. More complex situations could be handled at a slower rate.

The overall growth of C with resolution places an upper limit on object precision in any practical situation. In many cases performance could be enhanced if high resolution was maintained only on surfaces where it was actually needed. Nevertheless, many applications require much greater precision than would be practical for a simple octree system.

To accommodate such situations, a dual data structure approach is envisioned. A specialized data structure tailored for a specific application and its functional requirements would be used in conjunction with a general purpose octree-based system. This latter section would handle interactive geometric and geometry-related functions such as object manipulation, analysis and display.

Items in the application data structure would be relatively permanent whereas the octree data could be more or

less transient. This is somewhat analogous to the relation between a conventional 2-D geometric data base and the frame buffer memory of a raster graphics display. The description of a circle in the data base may be an actual part of a design while its representation in the frame buffer memory would be temporary data generated to fulfill a specific need such as visualization by the user.

Of course, frame buffer type data may be made part of the application data base. This could include the digitization of a real-world image or perhaps a scene that would be difficult or very time-consuming to recreate. In like manner, octrees acquired from CT scanners or the final results of a design session could be retained in the application data base in octree format.

The overall strategy is as follows. Objects or parts of objects not maintained in octree format are converted from user format beginning at the root and continuing down as needed by the currently running processors on a demand basis. In practice, all items involved in a user session are kept in octree format to some modest depth. This would typically be the local depth at which each obel would contain only one or very few primitives from the application data base. This corresponds to the locally optimal grid size in the variable-grid technique of Franklin [21].

Sufficient information is kept with the current leaf nodes quickly to generate the subtrees when needed. They are

soon discarded when their usefulness has passed.

What is to prevent the generation of all nodes to a low level whenever a user makes a request? The answer is to develop algorithms that require computation (request nodes) in a quantity related (in some sense) to the complexity of the immediate situation rather than the number of and complexity of all objects involved. In interference detection, for example, the number of nodes examined could be a function of the nearness of the objects. In hidden-surface display, the computation could be related to the actual complexity of the scene generated. The development of such algorithms was a major part of this research.

CHAPTER 3

OCTREE GENERATION

In addition to octree generation, an objective of this chapter is to begin developing a body of "tools" to be employed by algorithms for performing sub-functions within the implementation constraints (simple arithmetic, easy VLSI implementation, etc.). This will be a "bag of tricks" from which specific solutions will be drawn as the need arises. These tools perform specific, not general, functions but will, hopefully, be broadly useful over many algorithms. Lower-level tools will be combined to form higher-level ones to implement more sophisticated functions. At the lowest level, tools are the simple arithmetic operators. To the system implementor they correspond to specific hardware subsystems to be used in the construction of special-purpose hardware processors.

In order to motivate the tool development, it will be placed within the context of solutions to increasingly difficult modeling system functions. In most cases, solutions to the 2-D (or 1-D) problem will be presented first for clarity, followed by the extension to 3-D.

3.1 Algorithm Considerations

Many factors were considered during algorithm design and development. A few of the major ones will now be discussed.

The mathematical operations that algorithms can employ are severely restricted because of speed, cost, implementation and simplicity considerations. The permitted ("legal") operations are integer addition and subtraction, magnitude comparison, shifts, and data movements such as LOAD, STORE, stack PUSH, stack POP, queue INSERT, and queue DELETE. These legal operations form a set called simple arithmetic.

Solid modeling functions can still be performed, in spite of these restrictions, because of the design of the data structure. In almost all cases where a product (or quotient) is needed, one of the factors is a power of 2. The desired result can thus be generated by the process of shifting.

Two phases of algorithm operation are defined: setup and run. During the setup phase, a small number of unrestricted computations are allowed for processing user requests. During the run phase, the requested solid modeling function is performed over the octree objects. Only simple arithmetic is allowed. In mass property measurement, an isolated multiplication or division may be needed to compute an intermediate or final property value.

During algorithm design, maximum advantage was taken of the more or less standard techniques that have proved to enhance performance. This includes extensive use of parallelism and pipelining, avoidance of iteration, looping or unbounded situations, and so on. The classical computation-versus-memory tradeoff was generally decided in favor of extensive use of memory.

For most of the algorithms, two catagories of tree traversal sequences, depth-first and breadth-first, are possible. They correspond to two strategies for attacking problems. A depth-first algorithm generally traverses a tree downward from parent to child, returning to the parent when all lower nodes have been processed. Breadth-first traversal processes all nodes at one level before working the next lower level.

Depth-first traversal tends to be used when local information is required whereas breadth-first is employed when global information is needed. Depth-first operations typically use a stack, either directly or via reentrant code, to maintain tree location. Breadth-first information is passed from one level to the next in a queue.

3.2 Octree Generators

It is expected that high-speed conversion from various high-level application formats into octrees will be required. Perhaps the most obvious method for this is the brute force use of spatial enumeration. A full tree is first constructed with all possible leaf nodes at the desired lowest level. The input objects are processed with the leaf nodes corresponding to the interior voxels marked F. As noted by Requicha [56], conversion from any popular SMS format to spatial enumeration is straightforward. The tree is then simply reduced.

The obvious difficulty with this procedure is the huge memory requirement ($O(8^n)$ where n is the lowest level) and the associated processing time (all leaves must be accessed at least once).

A variation is to generate 2-D quadtrees representing orthogonal slices through the universe. They are converted to an octree (voxels on a plane) and then unioned together. All possible obels in the universe are still accessed but the memory required may be substantially reduced. This method was used to generate the medical octree objects from 2-D CT images shown in Figures 7.2 and 7.3.

In some situations the bottom-up conversion methods of Samet [68, 69] for quadtrees can be used. For the most efficient cases, the computations can be proportional to

object complexity. This runs counter, however, to the general strategy of converting from application format to octree format in a top-down manner on a demand basis. Better methods are needed.

A proposed solution is the use of specialized software or hardware processors called octree generators. As shown in Figure 3.1 these would be preloaded with the object parameters. The user of the data (the octree processor) would request node values. The generator maintains the state of the traversal in an associated stack or queue.

From a complexity viewpoint, the efficiency of an octree generator is a function of the false-P rate. This is the fraction of nodes marked P that will have a value of E or F after reduction. This is not considered to be an error because the obel has not been incorrectly determined. The calculation of the final value has simply been postponed, requiring additional work.

If the false-P rate is zero, all nodes are correctly determined the first time and the tree is identical to the reduced tree. If the obel values can be determined in constant time, the computations grow linearly with object complexity (C). If, for a typical user request, only parts of a tree are needed, computations could be expected to be linear (in some sense) in the complexity of the specific case.

Conceptually, an unsorted input object can be converted

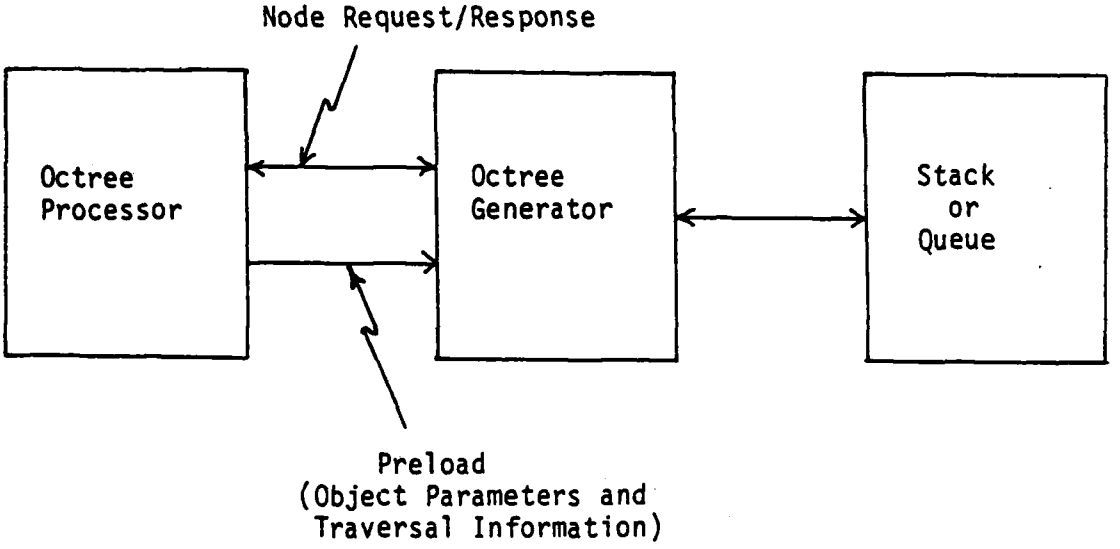


Figure 3.1 Octree Generator

into a sorted octree in linear time, rather than O(n log (n)) or worse time, because a radix-type sort over a finite "alphabet" (the obel locations) is involved [1]. The worse than linear growth of typical sorting operations is caused when comparisons between elements is required. None are required here.

The basic octree generation operation is to compare a test obel and the real object being converted. The result is one of the three status values, E if Obel $\cap$ Object = $\emptyset$, F if Obel $\cap$ Object = Obel, or P otherwise.

The octree generation strategy is as follows. Beginning with the root node the values of test obels in the output octree object are determined by comparison with the input (real) object. A node in the octree is created with this value. In most situations of interest this can be performed in constant time. E and F nodes are terminal and need no longer be considered. P nodes are subdivided with the corresponding children used as later test obels.

The first algorithm tool to be developed is the calculation of child vertex coordinates from the parent values. As shown in Figure 3.2 for 2-D, this is easily accomplished by means of additions and shifts (divide by 2). The 3-D or N-D equivalent is obvious.

The following sections discuss the conversion of convex objects. Conversion of concave objects is much more difficult and less well understood. Suggested approaches are

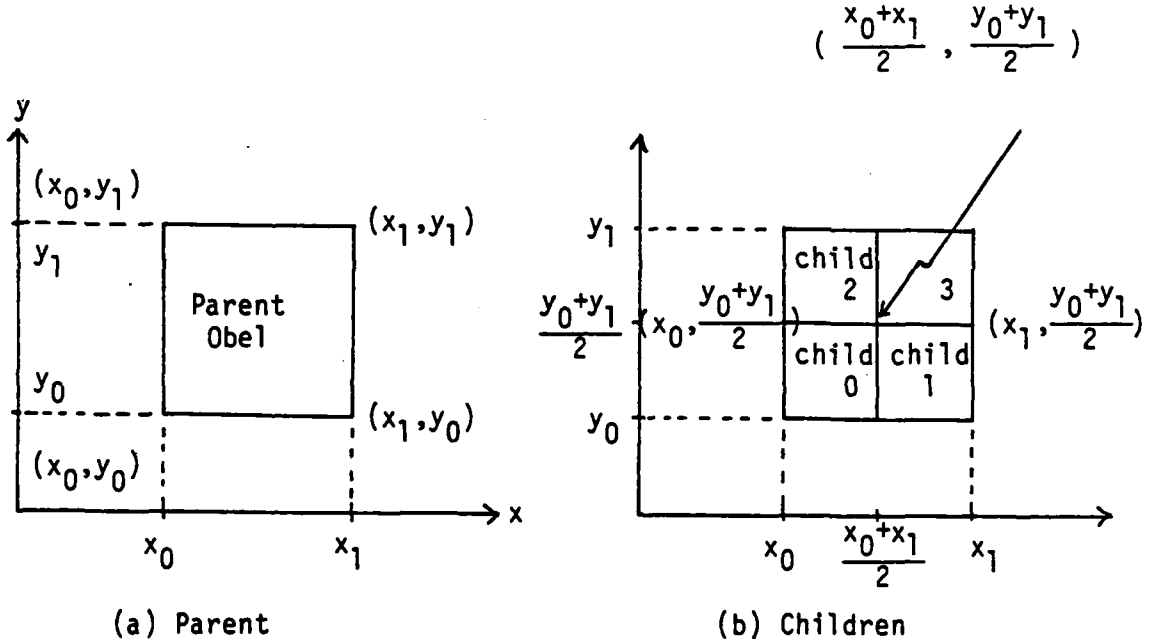


Figure 3.2 Calculation of Child Vertex Coordinates from Parent Values.

outlined in [43].

3.3 Orthogonal Blocks

The development of object generation tools will begin with a simple case, a 3-D block (rectangular parallelepiped) with faces perpendicular to the major axes.

A typical case is illustrated in Figure 3.3. The minimum and maximum coordinates in each dimension are noted. Three test obels with the three status values are shown.

Because the bounding planes and axes are orthogonal, the status determination can be decomposed into an independent comparison in each dimension. Determining of obel status is straightforward.

The development and analysis of the orthogonal block generation algorithm is presented in [43]. It has a zero false-P rate.

3.4 Convex Objects

The generation of octrees for convex objects is more difficult than for blocks because the dimensions cannot, in general, be analyzed independently. The basic low-level operation is determining whether a particular vertex point of a test obel is interior or exterior to the object. If the object is a convex polyhedron, the surface is described by a

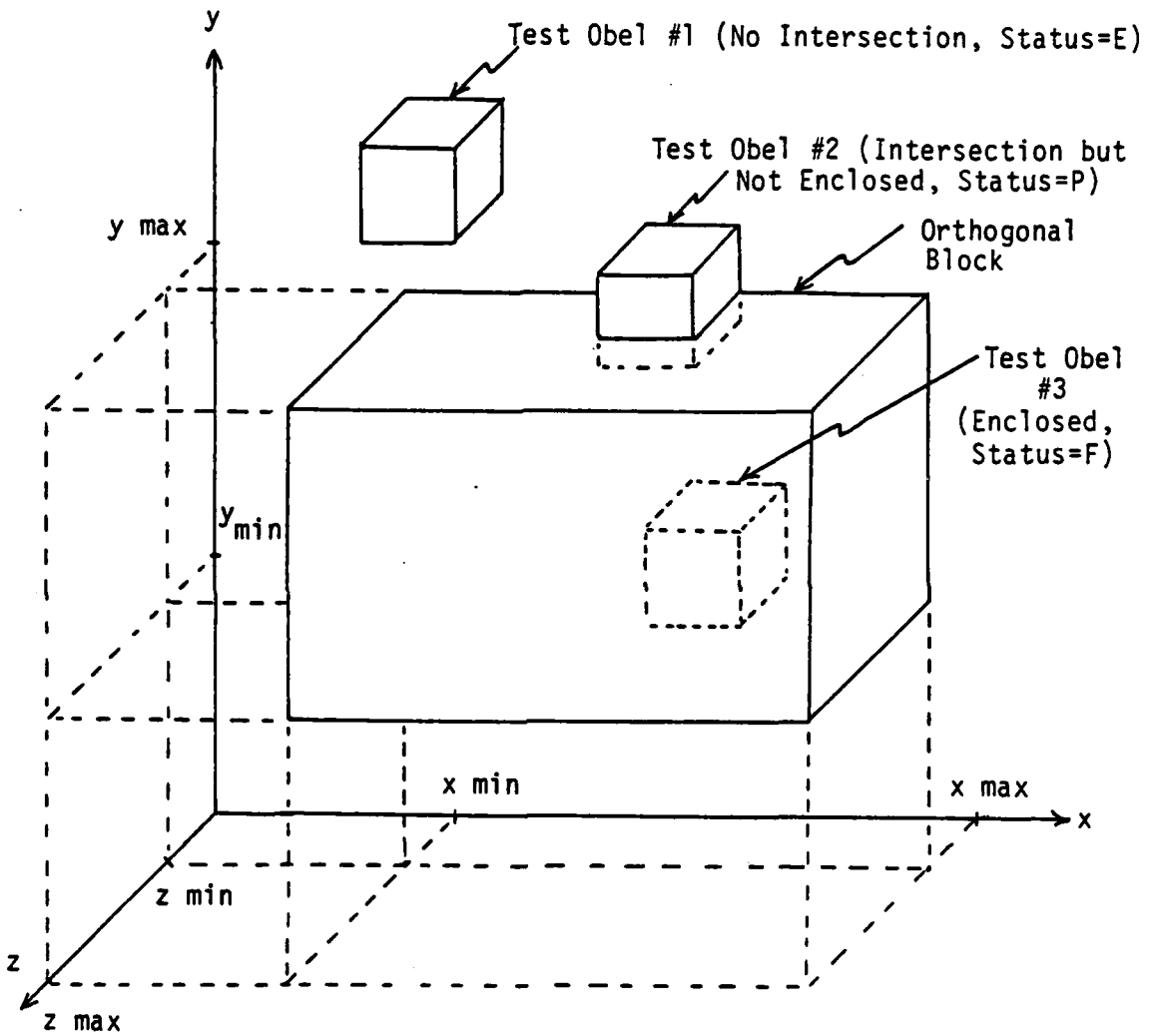


Figure 3.3 Octree Generation for 3-D Orthogonal Block

set of planar faces, each of the form:

$$Ax + By + Cz + D = 0 \qquad (3\text{-}1)$$

Each plane divides the universe into two half-spaces. The half-space containing the object will be called the positive half-space; the other is the negative half-space. For points not on the plane, (3-1) will evaluate to a positive or negative value depending on the half-space.

A simple substitution of the coordinates of the vertex point determines whether the location is on the plane or in one of the half-spaces. Note that a positive evaluation value does not necessarily indicate the positive half-space. Also, a point in the positive half-space is not necessarily within the object. The face plane is the entire plane whereas the face is a segment of the face plane.

Since multiplications are not allowed, the components of (3-1) are calculated for obel vertices by averaging from the parent components. A set of components for each vertex point in each dimension for each plane would need to be maintained. In 3-D this requires 24 values per plane.

Alternately, the signed perpendicular distance from the vertices of the universe to the plane can be calculated from:

$$d = (Ax+By+Cz+D)/((A^2+B^2+C^2)^{(1/2)}) \qquad (3\text{-}2)$$

The distance value for a new vertex in a child obel is the average of the two parent vertices defining the edge containing the new vertex. The sign of the distance

indicates the half-space. The new distances are calculated as follows:

$$d = (D\_VAL(AND(CHILD,VERT)) + D\_VAL(OR(CHILD,VERT)))/2 \quad (3\text{-}3)$$

where CHILD is the child number of the new obel, VERT is the number of the desired vertex of the new obel and D_VAL(n) is the distance of parent vertex number n. The AND and OR functions perform bit-wise Boolean operations on the child and vertex numbers in binary format.

For each plane, 8 values are used (one for each vertex). In practice, only one distance value need be kept. The remaining values can be generated by adding an offset from a pre-computed table.

For a convex polyhedron, if a point is in the positive half-space of all face planes, then it is interior to the polyhedron. If it is in the negative half-plane of any face plane, then it is exterior to the object. Otherwise, the point lies on the surface of the object (on one or possibly more faces). A surface point touches the object but is not considered to be interior to the object.

Thus, for polyhedral objects, an obel vertex point can be determined to be interior, exterior or on the surface using simple arithmetic without considering the location of the faces on the face planes or the actual intersections of face planes.

For convex objects defined by an analytic surface the

location of the point relative to the object can often be determined by evaluating the defining expression for the surface.

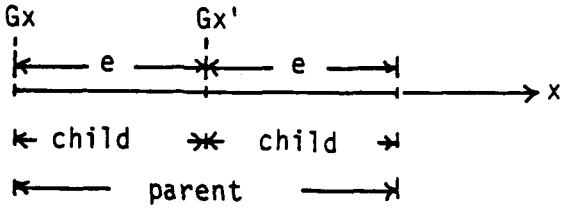
In 3-D, the most general second-degree equation is:

$$Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J=0 \qquad (3-4)$$

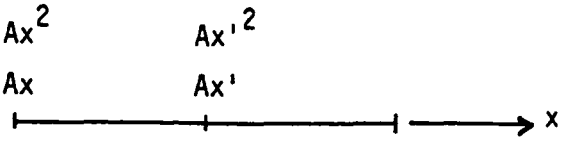
Note that the terms have three formats, a constant times a coordinate, a constant times the square of a coordinate and a constant times the product of two different coordinates. Generating the terms of the expression for the vertices of a child obel using simple arithmetic on parent values can be accomplished as illustrated in Figure 3.4. In 3.4(a) Gx is the product of coefficient G and the coordinate value for a vertex (x). The value of Gx' is the product of G and a new coordinate value (x'), a vertex of one of the children. The value of Gx' is desired as a simple function of Gx from the parent. Because the distance value between the two coordinates (x'-x) is equal to an edge of an obel at that level (e), the needed multiplication can be performed by shifts (e is a power of 2). It is computed as follows:

$$Gx' = G(x+e) = Gx+Ge = Gx+G(2^{(m-n)}) \qquad (3-5)$$

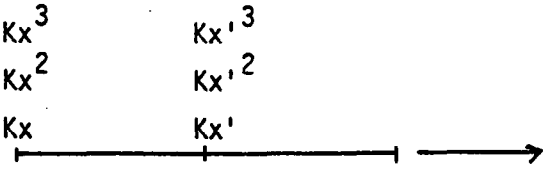
where m is the level of the lowest level in the universe (where e=1) and n is the level of the child obel. The first term is the parent value and the second is a shift of the constant G. Thus, a shift by m-n followed by an add operation will be sufficient.



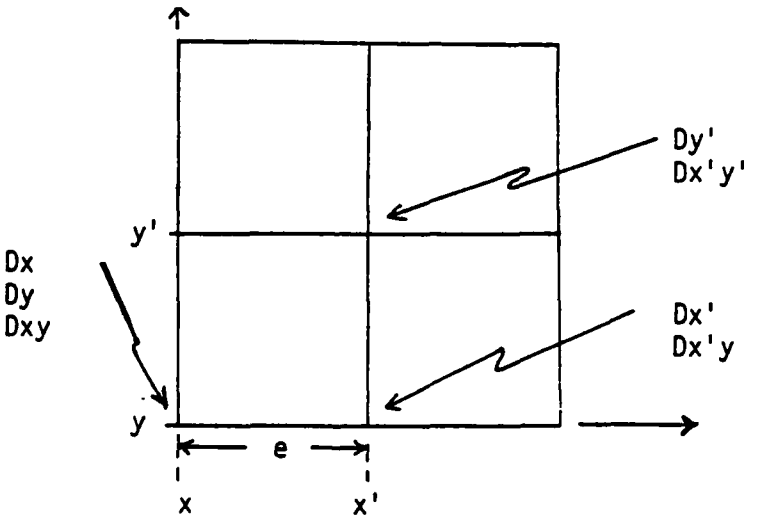
(a) Given Gx, calculate Gx'



(b) Given Ax and Ax^2, calculate Ax' and Ax'^2



(c) Given Kx, Kx^2 and Kx^3, calculate Kx', Kx'^2 and Kx'^3



(d) Given Dx, Dy and Dxy, Calculate Dx', Dy', Dx'y and Dx'y'

Figure 3.4 Generation Terms for Child Obel from Parent Values

In Figure 3.4(b) the value of Ax'^2 is desired from Ax and Ax^2. The value of Ax' will be needed later and is calculated as above. The Ax'^2 value can be evaluated by:

$$Ax'^2 = A(x+e)^2 = Ax^2+2eAx+Ae^2$$
$$= Ax^2+(2^{(m-n+1)})Ax+A(2^{2(m-n)}) \quad (3\text{-}6)$$

Two shift operations and two adds are required in addition to the shift and add to compute Ax'.

More sophisticated objects involving third-order equations require Kx'^3 as shown in Figure 3.4(c). The Kx' and Kx'^2 values are maintained as above with Kx'^3 calculated as follows:

$$Kx'^3 = K(x+e)^3 = Kx^3+2eKx^2+e^2Kx+eKx^2+2e^2Kx+Ke^3$$
$$= Kx^3+(2^{(m-n+1)})Kx^2+(2^{2(m-n)})Kx+(2^{(m-n)})Kx^2$$
$$+(2^{(2(m-n)+1)})Kx+(2^{3(m-n)})K \quad (3\text{-}7)$$

Five additional shifts and adds are used for a total of 8 each.

Two cross-product terms are illustrated in Figure 3.4(d), Dx'y and Dx'y'. The first is computed from the parent values of Dx, Dy and Dxy as follows:

$$Dx'y = D(x+e)y = Dxy+eDy = Dxy+(2^{(m-n)})Dy \quad (3\text{-}8)$$

A shift and an add are needed. The second is computed by:

$$Dx'y' = D(x+e)(y+e) = Dxy+eDy+eDx+e^2D$$

$$= Dxy+(2^{(m-n)})Dy+(2^{(m-n)})Dx+(2^{2(m-n)})D \qquad (3\text{-}9)$$

Three shifts and adds are needed.

Based on these tools, zero false-P algorithms for the conversion of polyhedra and restricted objects defined mathematically have been developed [43].

CHAPTER 4

ANALYSIS AND MANIPULATION

4.1 Object Properties

Of the following methods for object property determination, volume, surface area and separation of disjoint parts are 3-D extensions of published quadtree techniques [65, 66, 74].

4.1.1 Volume

Volume is the sum over all levels of the product of the F node count and obel volume (a power of 2) at each level. If an error tolerance is allowed, a minimum and maximum volume can be calculated breadth-first from the root. Only F nodes are summed into the minumum while P nodes are also summed into the maximum. When the average is within limits, processing is terminated.

4.1.2 Surface Area

Surface area is simply the sum of the areas of all exterior obel faces (faces separating an E node and an F node). F nodes may need to be subdivided (if a face touches both E and F nodes).

4.1.3 Center of Mass

If a homogeneous object is divided into n disjoint regions having volumes V_1, V_2, ... V_n and centers of mass $(x_{1,1}, x_{2,1}, \ldots x_{N,1})$, ... $(x_{1,n}, x_{2,n}, \ldots x_{N,n})$, the object center of mass, $(x_1, x_2, \ldots x_N)$ can be calculated as follows:

$$x_i = \sum_{j=1}^{n} (V_j x_{i,j}) / \sum_{j=1}^{n} (V_j) \qquad (4\text{-}1)$$

For an octree object, the disjoint regions are the F nodes. The centers are the obel centers and the volumes the obel volumes (a power of 2).

If a tolerance is allowed, a minimum and maximum can be computed during traversal.

4.1.4 Moment of Inertia

The moment of inertia, I, is defined as follows:

$$I = \sum_{i=1}^{n} (M_i R_i^2) \qquad (4\text{-}2)$$

where M_i is the mass of particle i and R_i is the perpendicular distance to the axis of rotation.

For a homogeneous octree, the mass of an F node is proportional to the volume (a power of 2). The distance squared value is computed using simple arithmetic as shown in the octree generation section, if the axis of rotation is (or has been made through rotation) parallel to a coordinate system axis.

The above formula assumes a point mass. Errors result if the center of a distributed mass is used. This is corrected by employing the parallel-axis theorem. The number of F nodes at a level is multiplied by the moment of inertia for an F node (at that level) about a parallel axis through the node center (a precomputed constant). The sum over all levels is added to the result of (4-2) to determine the exact value.

4.1.5 Segmentation of Disjoint Parts

The quadtree connected component labeling algorithm of Samet [66] is easily extended into 3-D to separate disjoint parts of an octree into multiple octrees.

4.1.6 Interior Voids

To obtain the number of interior voids the octree is negated (see below) and segmented into disjoint parts. The value is generally the number of parts minus one (the exterior).

A space filling operation is performed if one or more of the negated interior sections is unioned back into the original object.

4.1.7 Correlation

A measure of correlation between two objects is the fraction of the containing volume having the same status (E or F) in both objects. The containing volume is typically the bounding box of the union of the objects. The correlation can be expressed as follows:

$$(\text{volume}((\overline{A} \cap \overline{B}) \cap \text{BOX}) + \text{volume}(A \cap B))/\text{volume}(\text{BOX}) \qquad (4\text{-}3)$$

where A and B are the objects and BOX is the bounding box.

4.2 Set Operations

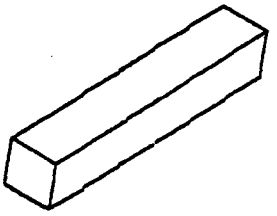
The set operations are the "regularized" operators [79]. Conveniently, this is the normal result of the quadtree and octree operations.

Algorithms to perform the set operations of union, intersection and difference have been published for quadtrees by Hunter and Steiglitz [29] and are directly extendable to 3-D octrees.

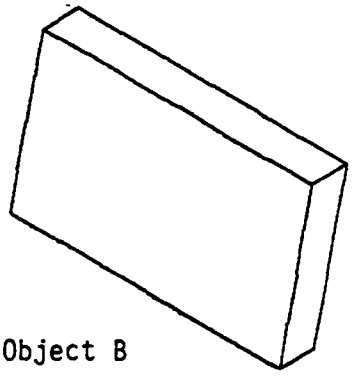
Computation is linear in object complexity (proportional to the sum of the C values for the input trees). Known algorithms for CSG and B-Rep are worse than quadratic because of the combinatorial explosion of face comparisons to detect new edges (see [28], for example).

The negation operation is performed by simply changing all F nodes to E and vice versa. Objects are sectioned by subtracting a "blanking object" (typically a half-space defined by a plane).

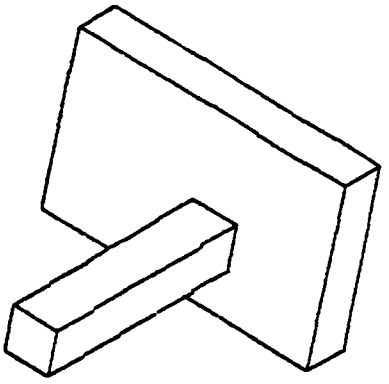
Figure 4.1 is an example. The set operations of union, intersection and difference are performed on objects A and B, resulting in three new octree objects.



(a) Object A

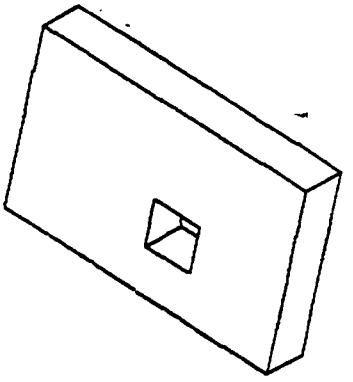


(b) Object B



(c) A U B

(d) A ∩ B



(e) B - A

Figure 4.1 Example of Octree Set Operations

4.3 Overlays

One of the most important tools employed within octree algorithms is the overlay. From an implementation viewpoint its importance would be difficult to overstate.

An overlay is a set of contiguous nodes at a single level, drawn from an input tree, such that the space it represents is guaranteed to enclose the space of an associated obel, the target obel, from another tree. The second tree is usually an output object being generated. By examining the overlay, a decision (perhaps an interim decision) can be made as to the status of the target obel.

For example, in 3-D an overlay containing a set of 8 obels all of which touch at a common point will always completely enclose a randomly located target obel of the same size or smaller if orthogonally oriented (not rotated). The record for each overlay obel may contain a pointer to a node in the input tree or the status value of an implied node taken from the terminal node ancestor.

The overlay can be thought of as a template containing a number of obels which cover a section of space. Within an overlay there is usually a single obel in a fixed spatial location (relative to the overlay) such that if the origin of the target obel is within it, the target obel is covered by the overlay. The shape of the overlay, the number of obels

it contains, the level of its obels relative to the level of the target obel, etc. depend on the specific algorithm.

By maintaining a set of active neighbors, the overhead of having pointers chasing through the tree to examine neighboring nodes is greatly reduced. From a computational growth viewpoint the savings are not significant because the expected value of the average traversal length required to locate a neighbor is a constant [66]. From an implementation viewpoint, however, the savings can be substantial because fewer memory references are needed.

Operating under the self-imposed restriction of using only simple arithmetic, the strategy in several of the algorithms below is to set up an output tree and then proceed to examine its obels for spatial interference with the input tree or trees. A status value based on the status of intersecting nodes is given to the output node.

Simple CSG systems perform an interference test by mathematically comparing the test primitive to each primitive in the object. The number of comparisons is equal to the number of primitives in the object.

Because of the spatially pre-sorted nature of the octree, it is not necessary to check all nodes in the input tree for interference. Only those which can possibly intersect the target obel, i.e., those in its overlay, are tested. Thus, the total number of tests needed to generate an output object is limited to the product of a fixed,

usually small, number (the number of nodes in an overlay) and the final value of C for the output tree (before reduction).

Advantage cannot be taken of this technique in most current SMSs because the primitives are not and usually cannot be spatially pre-sorted.

Several tools are needed to evaluate the status of the target obel. First, the overlay obels actually intersecting the target must be determined. This can be a difficult problem, depending on the algorithm.

Second, a status value must be determined. If all intersecting nodes have the value F, the target is given an F value. The same is done for E. Otherwise, the target is subdivided and each child processed in like manner.

Third, a sub-overlay must be generated for a specific child of the target. Each sub-overlay is identical in structure to the parent overlay with the new overlay obels being drawn from the set of children of the overlay obels.

Sub-overlay generation begins by determining the origin of the new target. Since it must be within a specific overlay obel, the exact enclosing child of the overlay is determined. The selection of the remaining sub-overlay obels is fixed by the structure.

4.4 Geometric Operations

For translation, scaling and rotation, the formal procedures are in [39].

4.4.1 Translation

The translation algorithm converts an object and a movement vector into a new tree representing the translated object. The process begins by generating an augmented overlay universe composed of the "old" universe containing the original object and a number of empty universes put together so that the "new" universe, which will contain the translated object, is covered by the old universe. The translation vector specifies the alignment.

Beginning with the root of the new universe, the new tree is traversed and its node values generated by simultaneously traversing the old tree using the overlay technique outlined above. If a terminal node in the new tree is generated, no descendants of that node need be considered. If an ambiguity exists and the status cannot be resolved, a P node is generated. Its children are produced in the same manner.

The overlay obels can be at any level relative to the target obel. The lower the level below the target, the more numerous but more accurate the result (lower false-P rate).

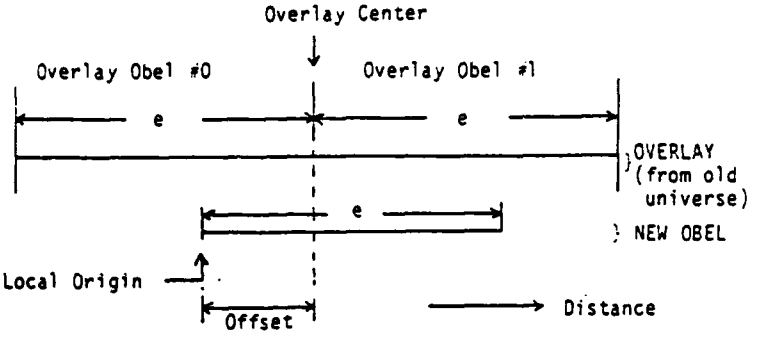
The translation algorithm presented here uses an overlay with 2^N obels at the same level as the corresponding new obel.

Figure 4.2(a) illustrates a 1-D overlay. The target has an edge size of e. The overlay is made up of two adjacent obels of the same size from the old universe connected at the overlay center. The offset value is the distance from the local origin (lower end of target) to the overlay center. It is limited to 0<=offset<e. The equivalent in 2-D is shown in Figure 4.2(b). The 8 overlay cubes and the target cube for 3-D translation are illustrated in Figure 4.3.

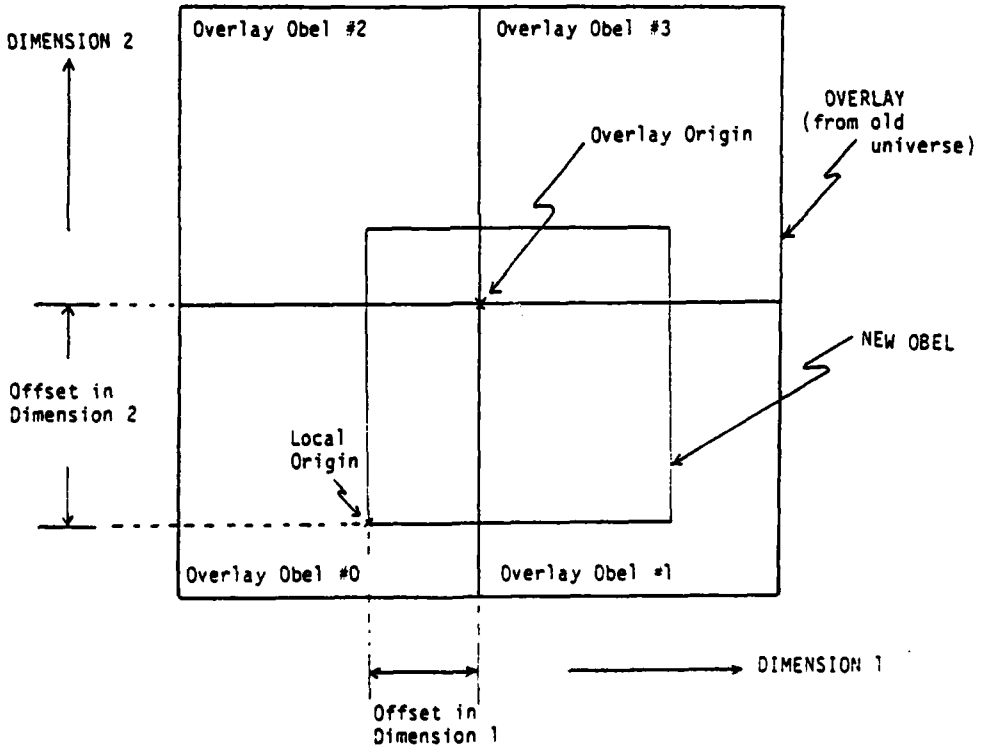
The orthogonal block techniques from the object generation sections above are used to determine intersection. The target obel corresponds to the convex object and the overlay obels correspond to the test obels. One difference is that the target obel does not remain fixed for all time. It subdivides. This presents no problem because the point-to-edge or point-to-plane distance values are easily updated for target subdivision using the techniques for test obel subdivision. The edges and faces of target obel children are parallel to the parent edges and faces.

A detailed description of the translation algorithm is given in [39].

An example is presented in Figure 4.4. An object and the six obels forming it are shown in 4.4(a) and 4.4(b), respectively. The new object in 4.4(c) was created by performing a translation in z by half the width of an obel.



(a) One-Dimensional Overlay



(b) Two-Dimensional Overlay

Figure 4.2 Overlay Structure

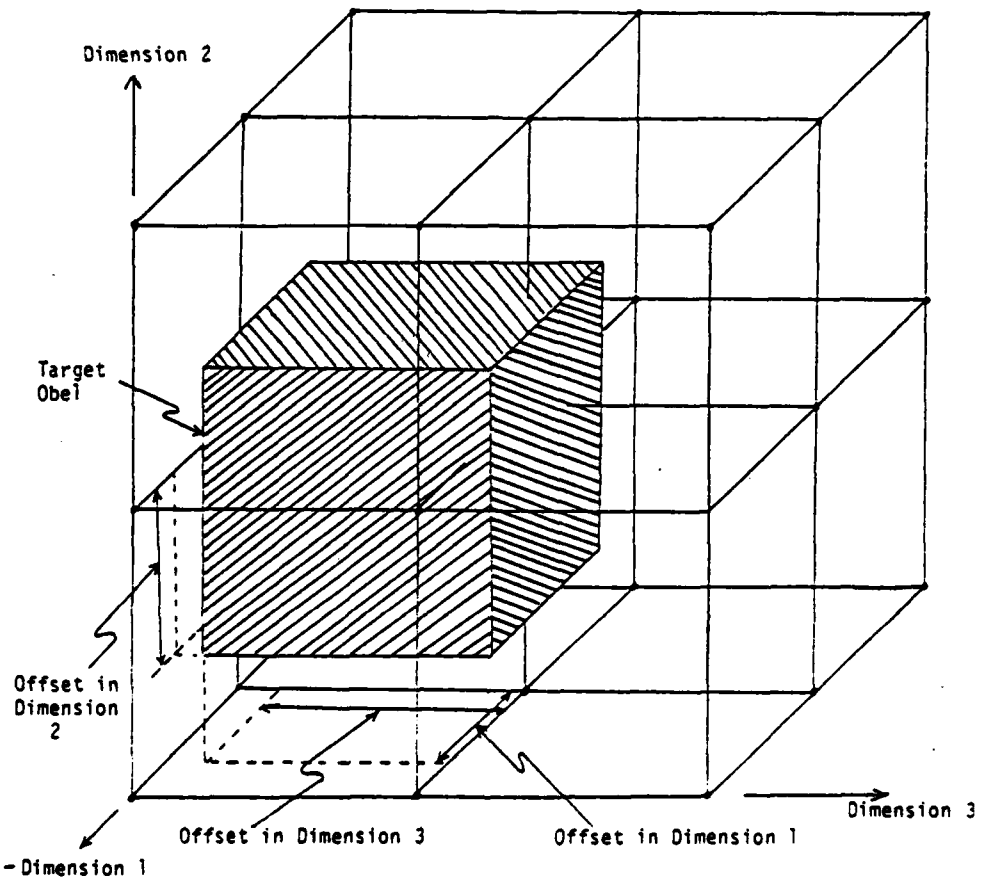
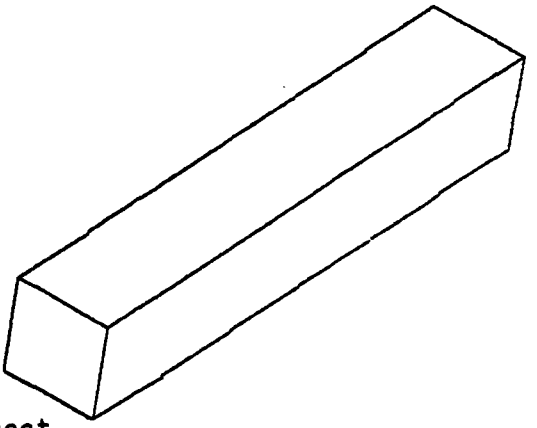
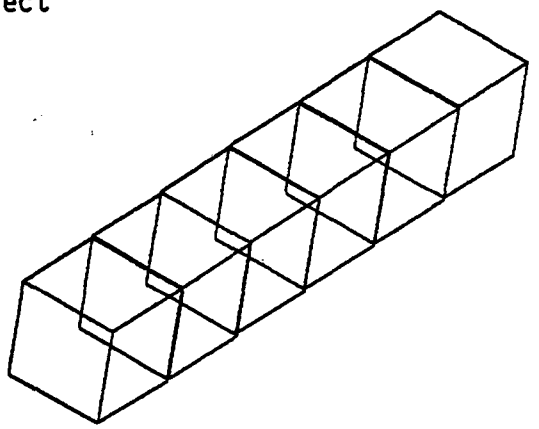


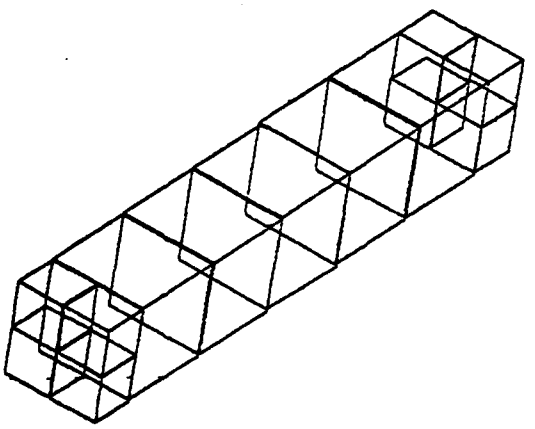
Figure 4.3 Three-Dimensional Overlay



(a) Object



(b) Object Obels



(c) Translated Object

Figure 4.4 Example of Octree Translation

Note that each end is now formed by four obels from the next lower level.

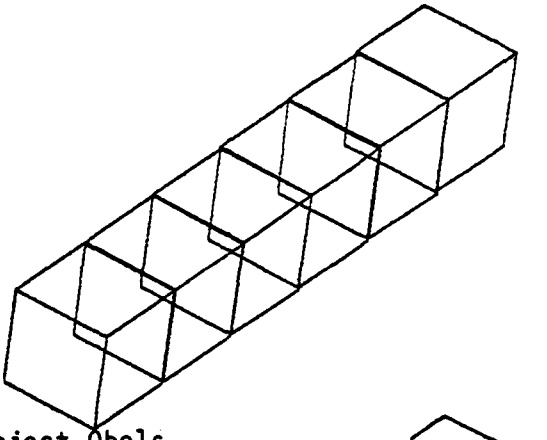
4.4.2 Scaling

Scaling an object by a power of two in all dimensions is accomplished by adding or deleting levels at the root. An object is halved in each dimension by adding one level at the top. The new root points to one branch node, the old root, and 2^N-1 empty terminal nodes.

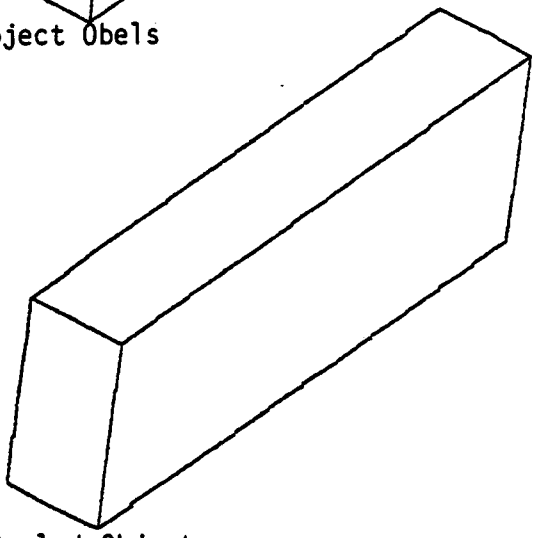
In like manner, selecting one of the first-level nodes to be a new root doubles the size of everything within it. For an arbitrary section of space, it is translated into a first-level obel and then expanded to fill the universe. Objects can be expanded or reduced by any power of 2 by, in effect, repeated expansion or reduction.

Scaling by a factor other than a power of 2 is accomplished using an overlay scheme similar to that used for translation. The target obel, however, may be smaller than the overlay in one or more dimensions. In addition, a single set of offset values cannot be used. The local offset vector must be computed independently for each child of the target obel. A detailed description is presented in [39].

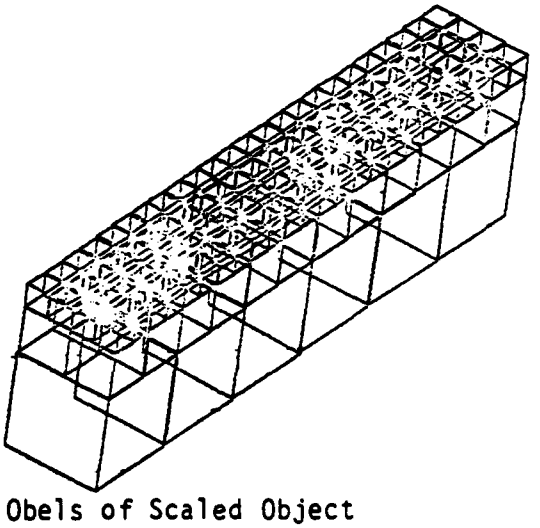
Figure 4.5 shows an object scaled by a factor of 1.75 in the y direction. The original object is in 4.5(a) and the new object in 4.5(b). In 4.5(c) the additional obels are



(a) Object Obels



(b) Scaled Object



(c) Obels of Scaled Object

Figure 4.5 Example of Octree Scaling

visible.

4.4.3 Rotation

Rotation by 90 degrees about an axis can be performed by a simple reordering of the nodes in the tree. For an octree, if the center of rotation is the center of the universe, rotation by 90, 180 or 270 degrees or reflection across a plane through the center parallel to a face of the universe or oriented at 45 degrees is accomplished by reordering or, within an algorithm, a change in the traversal sequence.

Figure 4.6 is an example of an object and two new versions rotated by 90 degrees using this method.

For rotation about an arbitrary point, the object can be translated to the center of the universe, rotated, and translated back.

Rotation by an arbitrary angle is more difficult. In 2-D, the overlay arrangement for a 0 to 90 degree rotation is shown in Figure 4.7. Nine overlay obels are required. The local origin of the target obel is always in overlay obel B. Clearly, for 0 deg. to 90 deg. rotation, the target will be covered by the overlay in dimension 1. In dimension 2, the maximum reach of the target is 1 + SQRT(2) which is less than 3, the height of the overlay. Coverage is thus guaranteed.

Again, the techniques from the object generation sections are used to determine overlay obel intersection. A

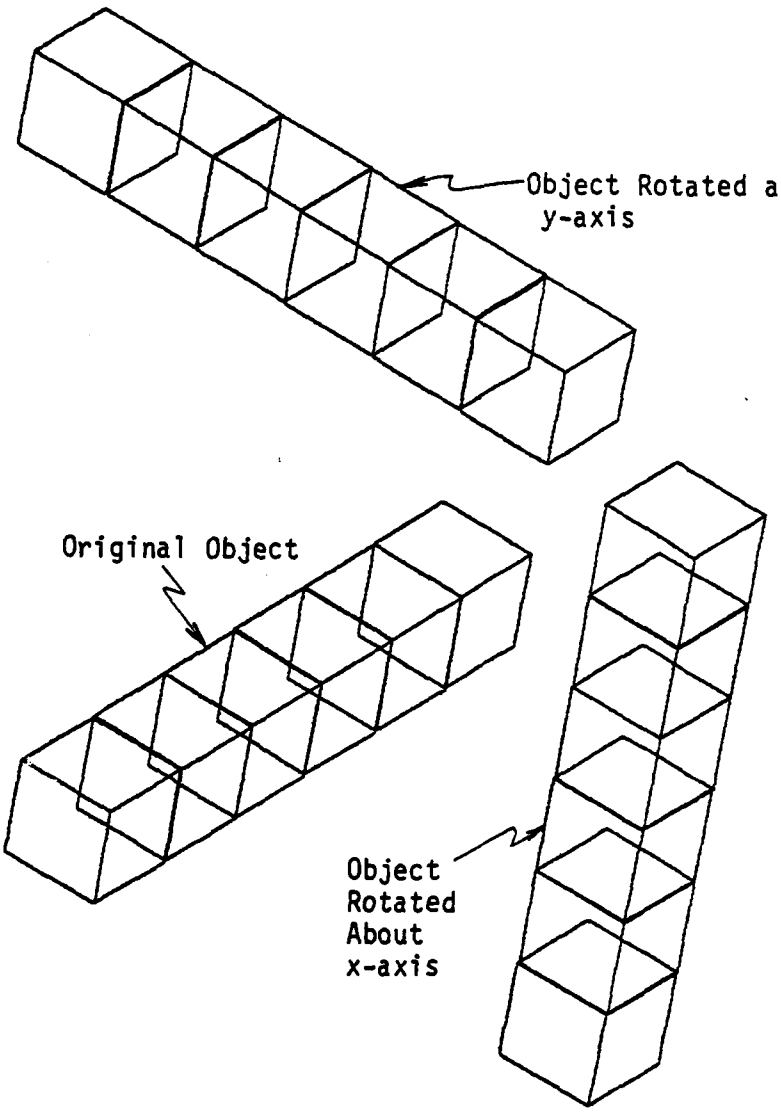


Figure 4.6 Example of Octree Rotation by 90°

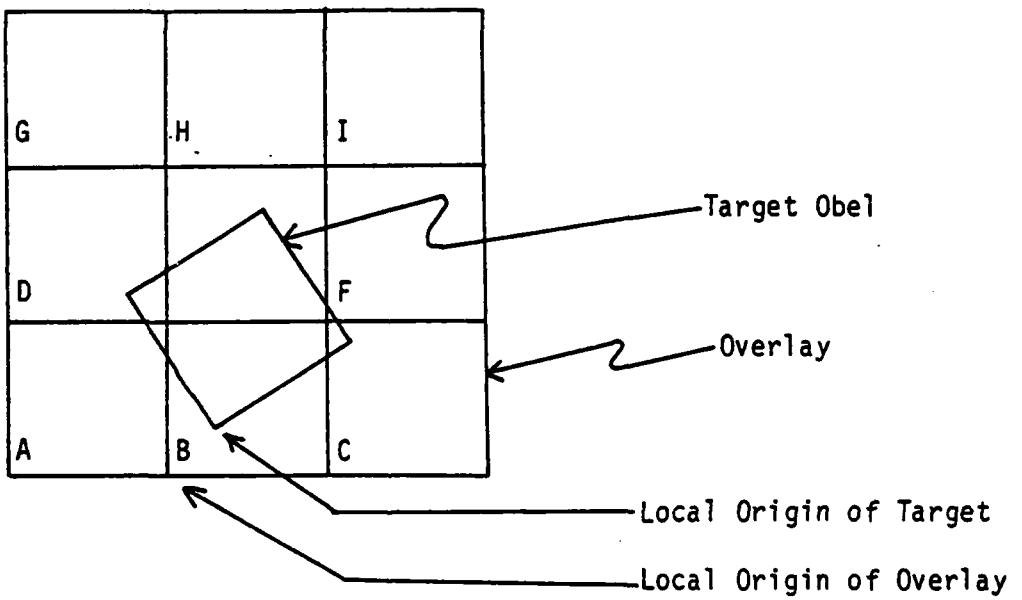


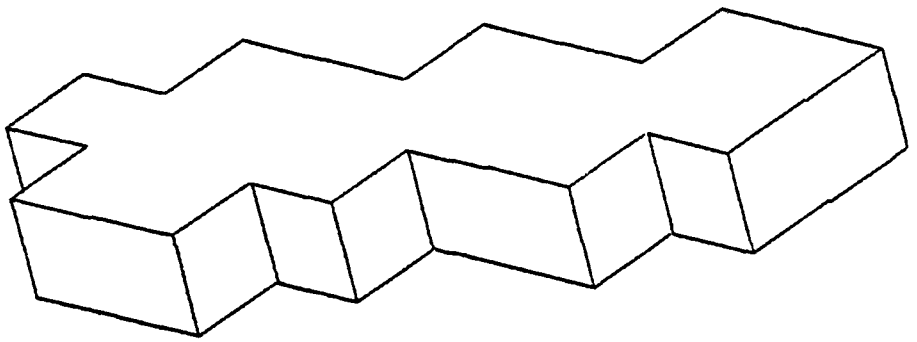
Figure 4.7 Rotation Overlay

detailed description of the algorithm is given in [39].

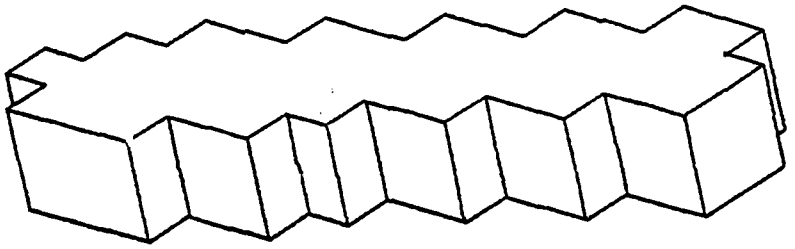
For an arbitrary 3-D object rotation, three operations could be performed using a slightly modified 2-D algorithm, one for each axis. An even simpler method is to perform multiple skew operations. Iftikhar [32] has proposed this based on the 2-D work of Catmull and Smith [15]. This approach may be generally undesirable, however, because at the lowest levels aliasing products are generated on the object surface when nodes are forced to be terminal. The object is essentially being redigitized from a previously digitized object. This tends to corrupt the surface and is compounded by the repeated operations that are required. This is minimized by computing nodes to lower levels, by performing operations in a single pass and by regenerating instances of an object from the original octree model each time it is moved or changed (rather than incremental movement of a single object).

Because of this, 3-D rotation in one pass is preferred. This is accomplished by extending the 2-D scheme into 3-D by the use of a 4 by 4 by 4 overlay. As shown in [43], a 3 obel per edge overlay is not sufficient.

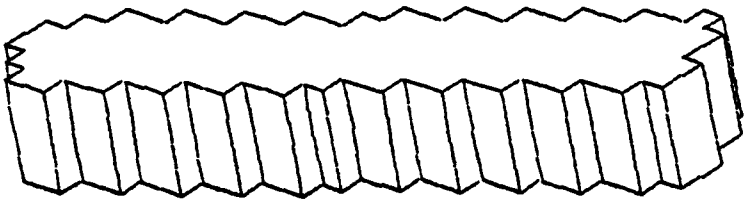
Figure 4.8 is an example of rotation by an arbitrary angle. The six-obel object from Figure 4.6 has been rotated by an angle selected at random (28.7 degrees). In 4.8(a), obels are forced to be terminal at the same level as the obels in the original object. In the algorithm used, all P



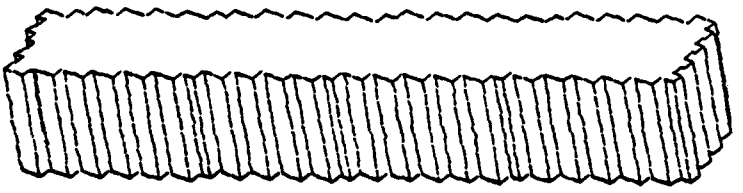
(a) Same Level



(b) Level-1



(c) Level-2



(d) Level-3

Figure 4.8 Example of Octree Rotation by Arbitrary Angle (27.8°)

nodes at the lowest level are given the value F, causing an enlargement of the object. In 4.8(b) through (d), increasingly lower levels are used, allowing a higher precision result.

4.4.4 Concatenated Geometric Operations

Using homogeneous coordinates [48] any number of sequential linear operations ($x' = A_1x + B_1y + C_1$, $y' = A_2x + B_2y + C_2$) can be concatenated and reduced to a single matrix of coefficients (3 by 3 for 2-D or 4 by 4 for 3-D). This, of course, includes the geometric operations of translation, scaling, rotation and skewing. The composite transformation can then be performed in a single matrix multiplication. The correlation can be expressed as follows for 2-D:

$$[x',y',1] = [x,y,1] \begin{bmatrix} A_1 & A_2 & 0 \\ B_1 & B_2 & 0 \\ C_1 & C_2 & 1 \end{bmatrix} \qquad (4\text{-}4)$$

Reversing for the moment the direction of data flow in an overlay operation (the target obel will now "generate" the overlay), the above concatenated transformation will be performed if a parallelogram-shaped target obel is used as shown in Figure 4.9. The target universe is assumed to be of unit edge length. The matrix coefficients determine its location, orientation and shape.

The C coefficients specify the location of the origin of the universe. The A coefficients determine the length and

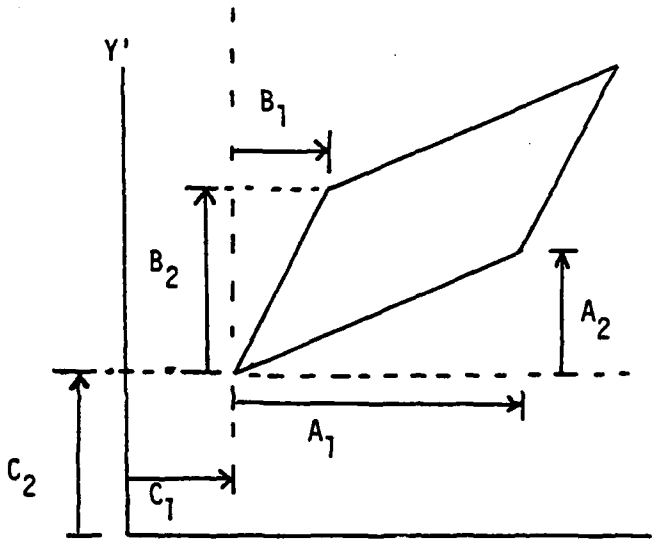


Figure 4.9 Target Obel for Concatenated Geometric Transformation

slope of the lower and upper edges relative to the overlay whereas the B coefficients determine the left and right edges. The original universe containing point (x,y) is shown in Figure 4.10(a). In 4.10(b), the transformed x value is shown. The C_1 value gives the origin offset. The x value specifies the fraction of the lower edge of the target universe spanned between the origin and the projection of the point on that edge. Since the projection of the edge on the x' axis is A_1, this displacement, when projected on the x' axis is A_1x.

The value of y is similarly the projection on the left edge which projects a distance of B_1 on the x' axis. The x' component due to the value of y is thus shown to be B_1y. The value is the sum of the three components or $x' = A_1x + B_1y + C_1$. The value of y' is computed in like manner as $y' = A_2x + B_2y + C_2$.

For a universe with a non-unit edge, the matrix coefficients are simply multiplied by the edge value to generate the target universe parameters.

The matrix product for concatenated linear transformations in 3-D can be expressed as follows:

$$[x',y',z',1] = [x,y,z,1] \begin{bmatrix} A_1 & A_2 & A_3 & 0 \\ B_1 & B_2 & B_3 & 0 \\ C_1 & C_2 & C_3 & 0 \\ D_1 & D_2 & D_3 & 1 \end{bmatrix} \qquad (4\text{-}5)$$

The geometric values for the target obel used to perform equivalent transformations are shown in Figure 4.11.

AD-A132 472 THE OCTREE ENCODING METHOD FOR EFFICIENT SOLID MODELING 2/2
(U) RENSSELAER POLYTECHNIC INST TROY NY IMAGE
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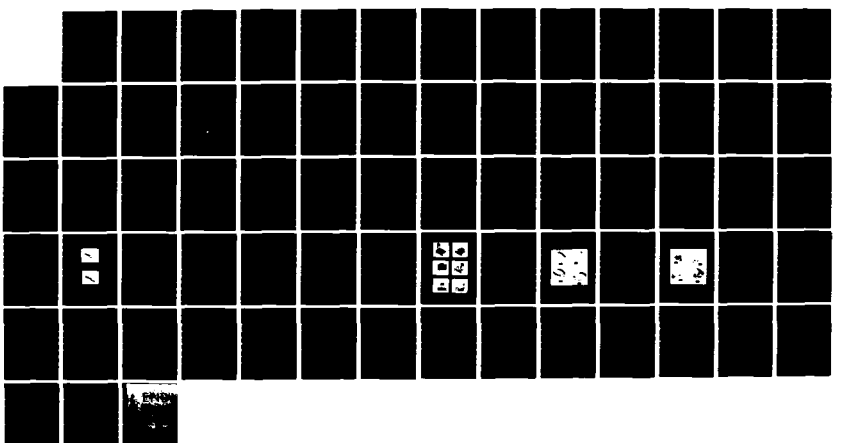
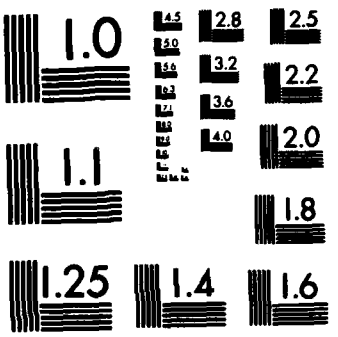
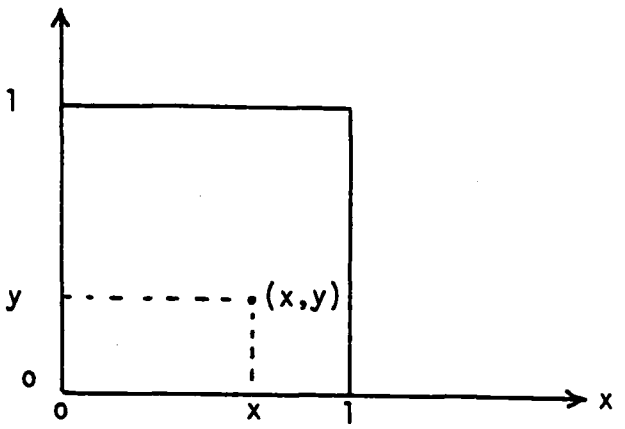
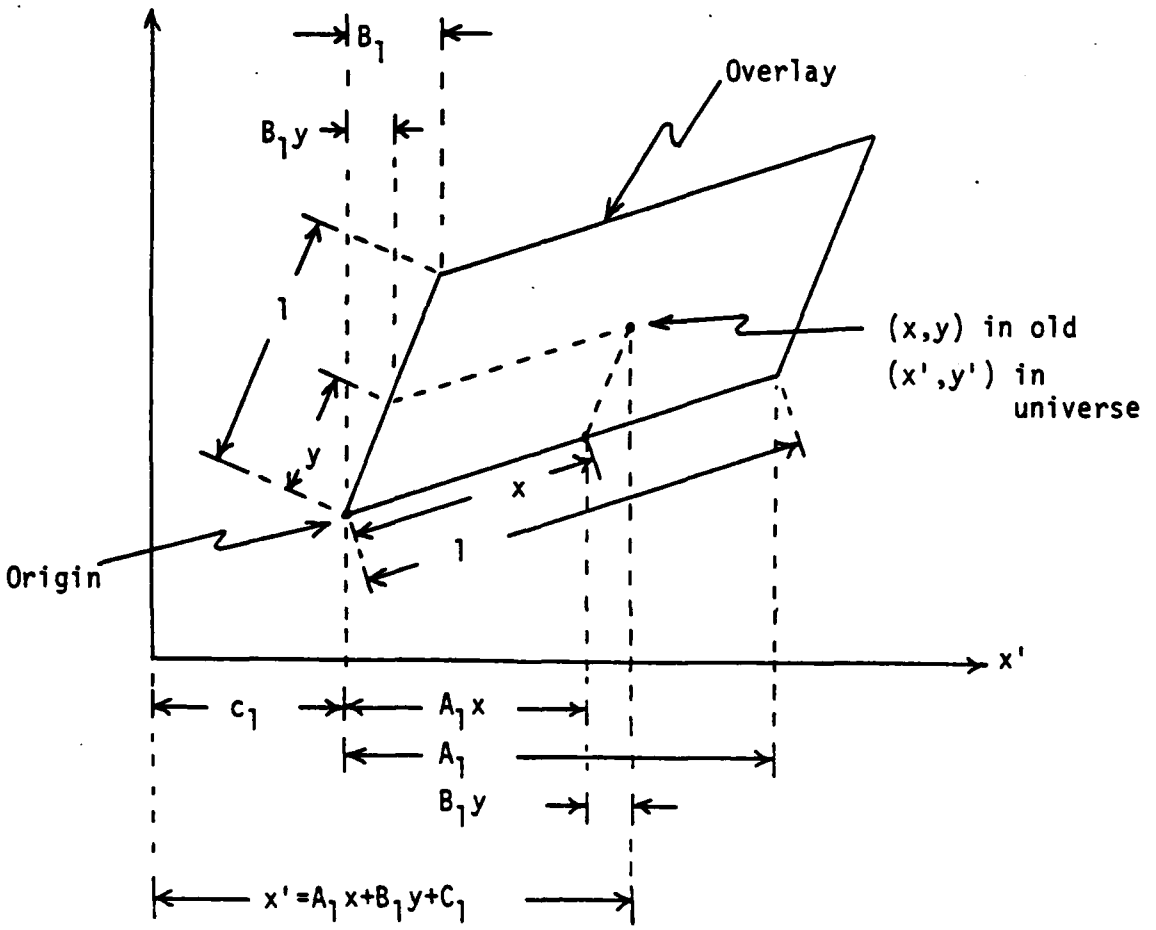




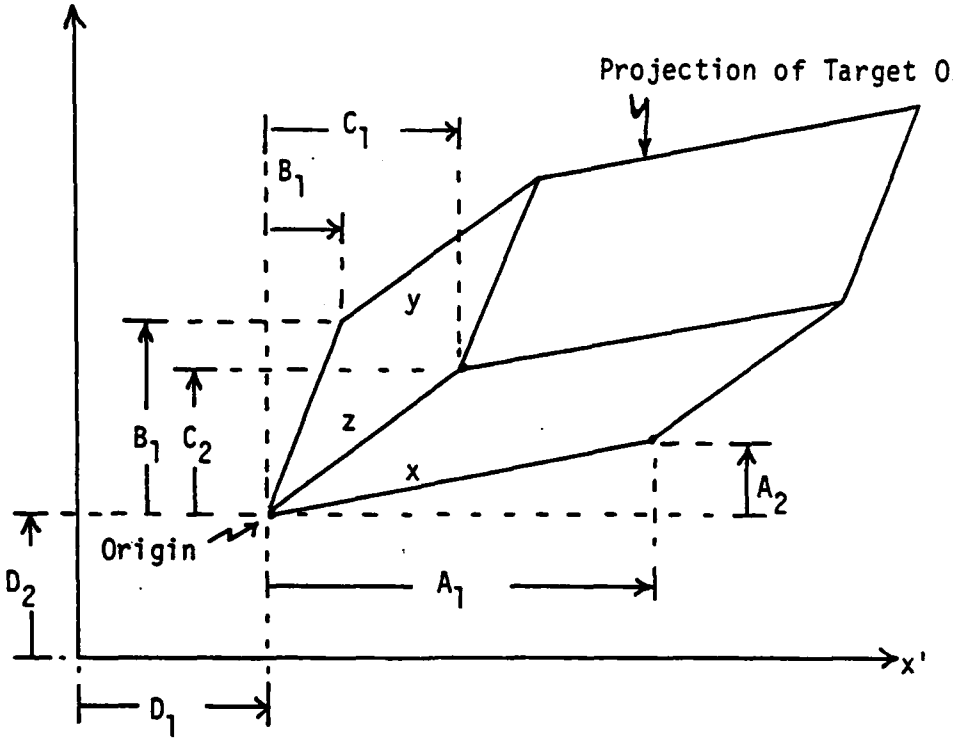


(a) Universe Before Transformation

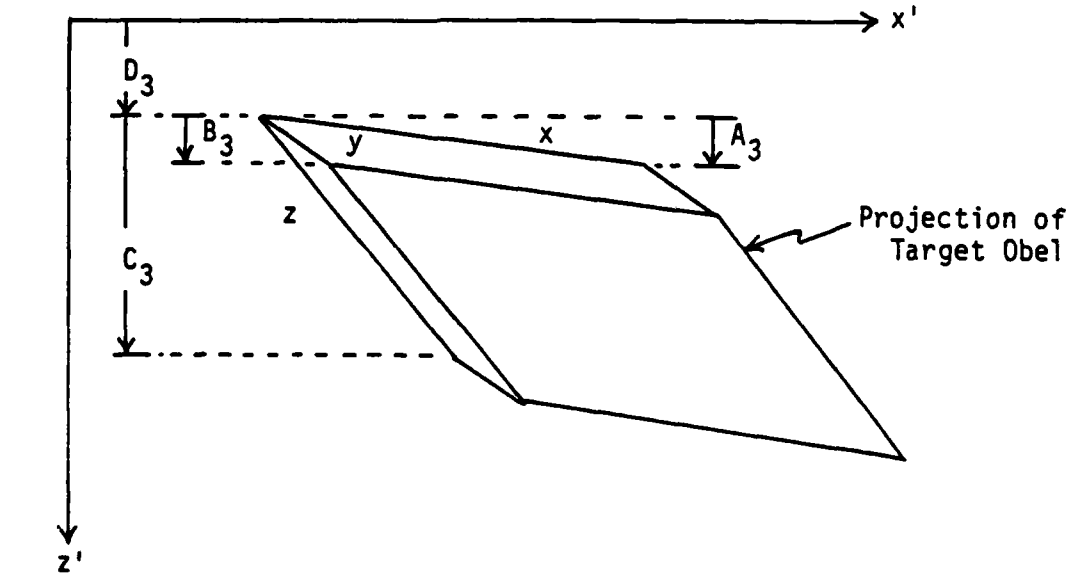


(b) Generation of Transformed x Value

Figure 4.10 Transformed Coordinate Value



(a) Projection of Target Obel on x'-y' Plane



(b) Projection of Target Obel on x'-z' Plane

Figure 4.11 Target Obel for 3-D Concatenated Geometric Transformation

Returning to the original data flow from overlay to target obel, the new obel is generated from the overlay when it is deformed according to the inverse of the coefficient matrix.

The distance values needed to determine overlap can be updated using previous techniques because the edges and faces of the children are parallel to those of the parent.

If the relative tree level difference between the target and overlay is fixed, the number and arrangement of overlay obels cannot be fixed unless restrictions are placed on the matrix coefficients. An alternate strategy is to fix the overlay configuration and subdivide the target obel at algorithm initialization until coverage is guaranteed.

If the shape of the target obel is generalized (not restricted to a parallelogram) nonlinear transformations such as perspective deformation can be performed [43].

CHAPTER 5

INTERFERENCE ANALYSIS

5.1 Interference Detection

Interference detection as used in this section is the detection of a situation in which the proposed insertion of a "new" 3-D object into a workspace at a specific location would cause it to occupy space already occupied by an existing "old" object or objects. It may be sufficient simply to determine that an interference exists. In other cases, the actual spatial interference "object" may need to be computed.

For a practical system, two primitive operations are required, object insertion (including interference detection) and object deletion. Additional operations could be defined.

An interference universe is maintained in which large numbers of objects can be inserted and deleted efficiently. Since it can be assumed that some objects will be very complex, it would be very desirable for the required computations to be related to situation complexity, not object complexity.

Conventional interference detection as performed by existing SMSs takes the following form [80]:

$$\bigcup_{i=1}^{n} (\text{NEW\_OBJECT} \cap \text{OBJECT}_i) = \emptyset \qquad (5\text{-}1)$$

where NEW_OBJECT is the object to be inserted, OBJECT_i is the i-th object already in the interference universe and n is the number of objects.

Clearly, such a procedure could require a processing time proportional to the product of the complexity of the new object and the complexity of all existing objects in the universe.

For schemes such as CSG and B-Rep, the unioning of all existing objects into a single object for testing would probably not reduce the problem because most if not all of the primitives would typically be retained. In any event, if some method were used to reduce the primitive count, the deletion problem would become more difficult. The subtree corresponding to a particular object could no longer be simply deleted.

The use of a binary tree to detect the intersection of orthogonal rectangles has been reported by Bentley and Wood [5]. Briefly, a 1-D hierarchical "segment tree" is maintained for the width of rectangles while the universe is scanned in height. The sorted data format of the segment tree is shown to allow insertion and deletion in log time. The spatially pre-sorted nature of the octree will now be used to facilitate 3-D interference detection.

Two data structures are defined, an object table and an

octree interference universe. The object table will be indexed by object number, a transient number assigned to objects. Table information will identify the object, point to the tree location in memory, etc.

It will be assumed that all trees will be reduced. Trees with false-Ps could be used with slightly modified algorithms but efficiency would be reduced.

The interference universe must be configured such that objects can be distinguished at the obel level. Each node maintains a list of records. They contain the object numbers of interference objects having P or F nodes corresponding to the interference universe node and associated information such as tree pointers. P node records will be marked as expanded or unexpanded. They are expanded if their children are also contained in the interference universe. If their children have not yet been added to the lists attached to lower nodes in the interference universe, they are unexpanded nodes.

Nodes of the new object are inserted in a generally breadth-first traversal. The state of node insertion is saved in a queue. If an input P or F node is inserted into an interference node containing no objects (the list is empty), no interference can exist at this node or below. It is not expanded into lower levels.

If an input P node hits an F node or an input F node hits a P node in the tree, an interference has been detected

(all trees are reduced). If all nodes are P, they are expanded. This continues until an intersection is found or all potential interference nodes are expanded. The depth of the descent and, therefore, the number of nodes expanded is related to the nearness of the possible interference objects over the surface of the input object.

Deletion is performed by traversing the object's octree and deleting its records from the node lists.

A major problem is the expansion of memory to hold the interference universe. It could expand to eventually approach the sum of the C values for all objects. The vast bulk of the nodes would have been expanded in an ad hoc manner to detect specific close approaches; the probability that most would ever be used again would generally be low.

Obviously a "garbage collection" function is required to delete unneeded interference nodes and their lists. From the lowest levels, nodes would be deleted from the lists and the parent entry changed from expanded to unexpanded. Assuming node expansion is relatively costly in time, and insertions are not completely random in location, it would be advantageous not to perform this operation after every insertion. Spatially close inserts later may be able to take advantage of earlier expansions. An asynchronous process would perform the garbage collection based on depth and time since last use, similar to a cache memory in a conventional mainframe computer. User requests would, of course, take

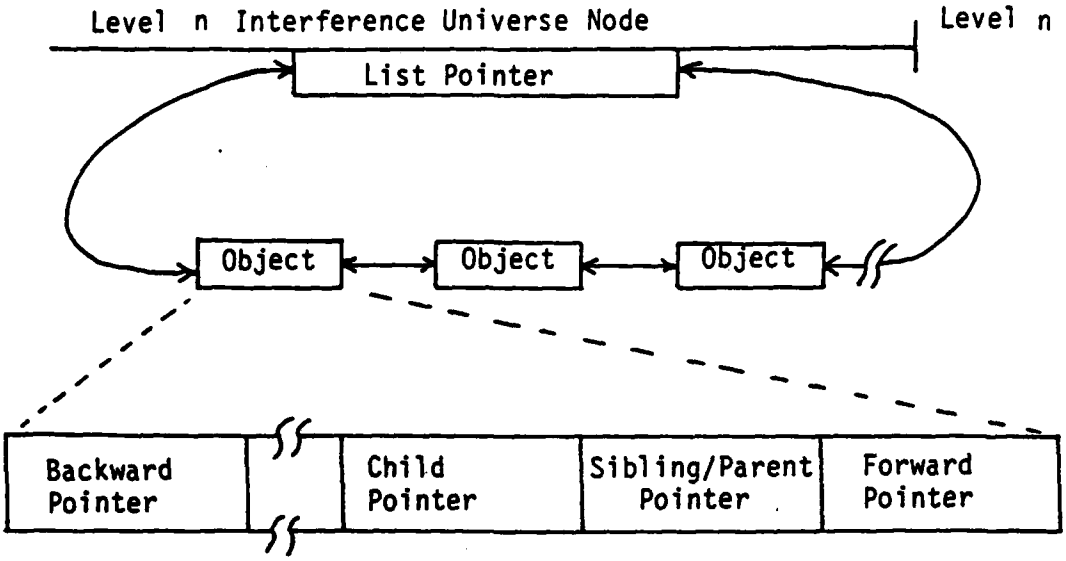
higher priority. Data base lockout (required to maintain data integrity) would only occur for very short periods while the children are deleted and the parent marked unexpanded.

The list search problem will now be considered. For large numbers of objects, a considerable effort could be expended searching object lists attached to nodes for expansion and deletion. This could add a term proportional to the product of the number of objects and the number of nodes processed, in the worst case.

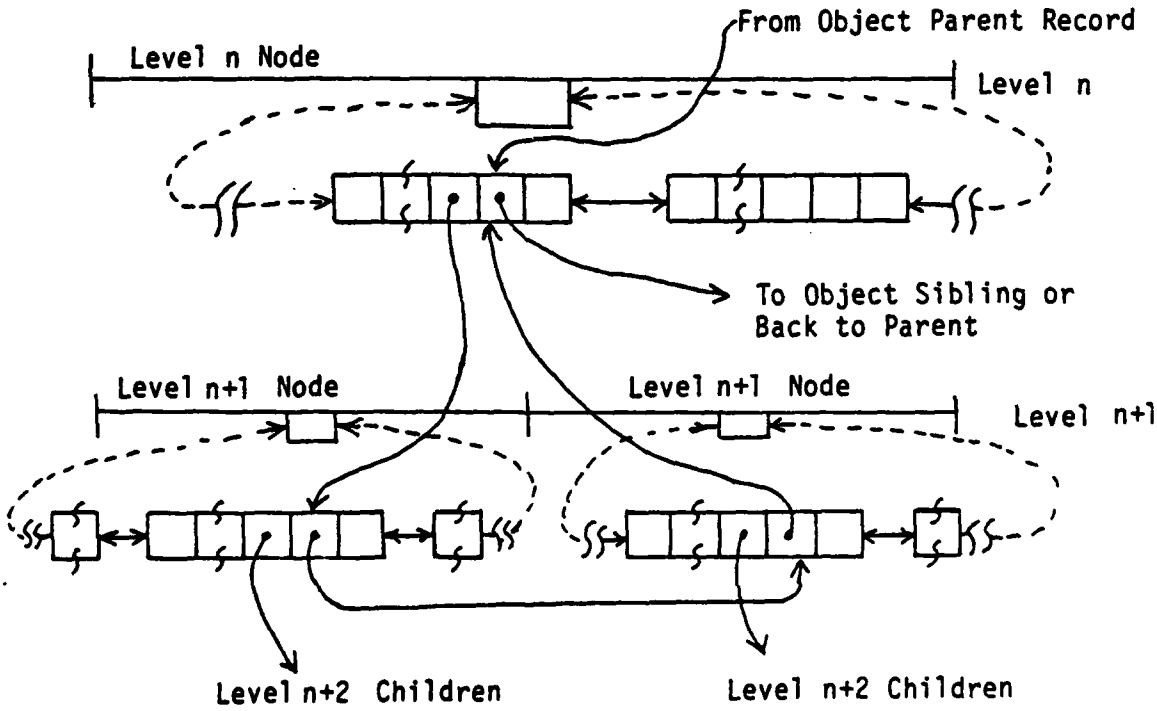
Two steps are proposed. First, separate lists are maintained for expanded nodes and unexpanded nodes. Second, object lists are multiply-linked as shown in Figure 5.1(a). Object records can be easily inserted, deleted and transferred between lists. Since all unexpanded nodes must be expanded in order to perform a test, no searching is required for object insertion. They are expanded then transferred to the expanded list.

The format of each object record is indicated in 5.1(a). In addition to forward and backward pointers, fields are provided for a child pointer and for a sibling/parent pointer.

Use of the data structure is illustrated in the Figure 5.1(b) for the 1-D case. The child pointer locates the record for the first child. The sibling/parent pointer locates the next sibling in order (E nodes are not represented) and eventually back to the parent.



(a) Object Record Format (Expanded List)



(b) Child and Sibling/Parent Links

Figure 5.1 Interference Detection Universe Pointers

These pointers can be used for garbage collection. First the data base is locked. The lowest level, lowest numbered child is selected. It and its siblings are deleted. The parent is then marked unexpanded and the data base released.

For deletion, the object table entry for the object is marked inactive. For each record processed, the table would be checked. If inactive, it would be ignored. In a specialized processor, this could probably be performed transparently with little loss of performance. An asynchronous process would then proceed to delete the object's records. When done, the object number is released for future use.

For insertion, if only expanded nodes are allowed or if no unexpanded nodes are encountered, the computations will be proportional to the number of input nodes that required processing. This number is related to situation complexity. The worst case would require the processing of a number of nodes equal to about C for the input object. The minimum would require almost no computation. For example, inserting a complex object into an empty octant of the universe would require negligible work even if the other octants contained objects of great complexity. If unexpanded nodes are encountered, expansions equal to C times the number of objects can be used to obtain an upper bound.

The lowest level at which nodes need be tested can be

determined if no intersection is found. For two objects to occupy some part of the same obel, they must be within SQRT(3) times the edge size of the obel. Thus, the lowest possible level will be:

$$n = \log_2(\text{SQRT}(3)E/d) \qquad (5\text{-}2)$$

where E is the length of an edge of the universe and d is the shortest distance between the two objects.

Based on the assumption of one-half branch nodes, the expected number of nodes tested would be $(2/3)4^{n+1}-1$. Empty nodes are included because they are accessed, even if not processed.

These upper bounds are only reasonable if all nodes must be divided down to the lowest level, that is, if the closest distance of approach to another object existed over the entire surface of the object.

If the object surface area in the vicinity of the point of closest approach is small and this distance is small compared to the equivalent distance over the remainder of the object, one could expect that, below a certain level, few branches would continue from one level to the next lower level.

If no interference is found, no conclusion can be drawn concerning the minimum distance of separation. In fact, the objects could be touching.

If a safety margin is needed around an object,

region-growing techniques could be employed or two interference universes could be maintained and tested, one having a coordinate system offset relative to the other.

The technique can also be used with hexadecatrees for kinematic analysis.

5.2 Swept Volume

In applications such as the verification of NC programming, collision avoidance and trajectory planning in robotics systems, interference analysis involves the "object" swept out by an object in motion.

Two algorithms have been developed for swept-volume determination. The first is the "translation" swept-volume algorithm. It generates the swept volume for an object translated in space while maintained in a fixed orientation. It is presented below. In the second, the "rotational" swept-volume algorithm, the object is rotated in a fixed body movement about a line in space parallel to an axis. It is described in [43].

The translational swept-volume algorithm accepts an octree and a curve in space (represented by a chain code) then generates the swept octree object.

5.2.1 Convolution Formulation of Swept Volume

In image processing, a 2-D convolution summation [14, 60] such as the following is often employed:

$$v(k,m) = \sum_{i=0}^{k} \sum_{j=0}^{m} u(i,j)h(k-i,m-j) \qquad (5\text{-}3)$$

where u is the input, h is the 2-D weighting function and v is the output.

In (5-3) h(k-i,m-j) is the weight at the location k-i units in the negative x direction and m-j units in the negative y direction from the output point being considered, (k,m). This corresponds to a point i,j from the origin and is therefore multiplied by u(i,j).

The translational swept-volume operation can be formulated in terms of a convolution operation. Again for simplicity, 2-D swept areas rather than 3-D volumes will be considered. The result is easily applied to 3 or more dimensions.

First the non-hierarchical situation will be examined. The input is a binary 2-D object contained in a spatial enumeration array (not a hierarchical tree). An E value indicates exterior and an F value, interior. The weighting sequence will be a similar array with an F value at each location that would be visited by the square at location 0,0 if stepped along the sweep path. All other weightings are zero.

The (5-3) convolution formulation is rooted in the 1-D processing of functions of time. The weighting sequence is defined from time 0 in a positive time direction. In two and three dimensions, such directional distinctions seldom exist. In order to be consistent with previous usage, universe coordinate values will be kept positive. This requires that the above sense of direction be reversed. Objects will be swept along a path that is entirely within the quadrant containing negative coordinates relative to the origin. The index values i and j will thus run from k and m, respectively, to n, the highest array index. In a real system, of course, the summation index would be expanded to handle both positive and negative coordinates.

In the swept-area case, the sum from (5-3) indicate the number of steps along the path for which the location was occupied but is typically a useless piece of information. A single occupancy of the location is sufficient. Thus, the binary input and binary weighting functions can be processed by the following version of convolution (F = TRUE, E = FALSE):

$$v(k,m) = \underset{i=k}{\overset{n}{OR}} \; \underset{j=m}{\overset{n}{OR}} \; AND(u(i,j),h(k-i,m-j)) \qquad (5\text{-}4)$$

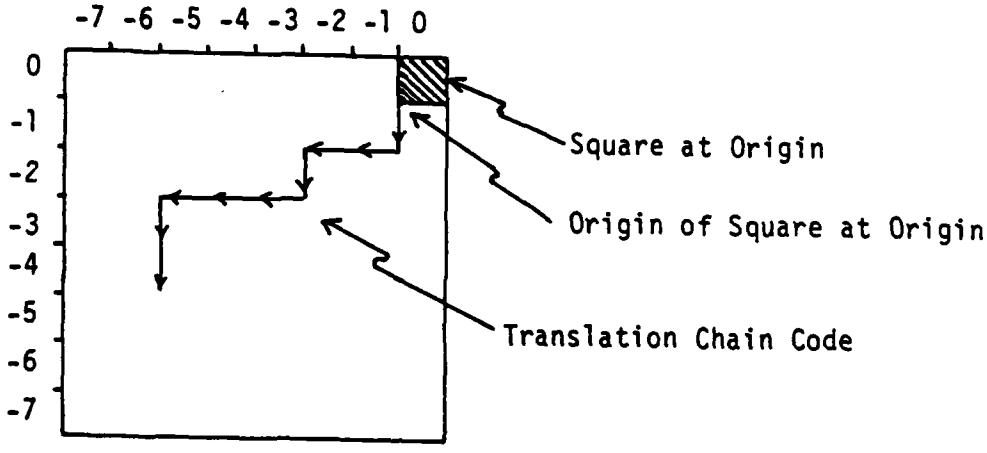
In operation, the calculation can be terminated when the first true value (occupancy) has been found.

5.2.2 Example of Swept Volume (Non-Hierarchical)

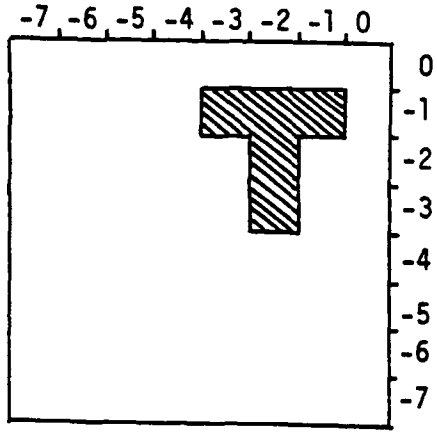
An example will serve to clarify the concept. In Figure 5.2(a) a translation curve represented by a 4-direction chain code [24] is shown. It is independent of location but for convenience is anchored to the origin of the coordinate system. A T-shaped object is shown in 5.2(b). The swept area of the object for the sample sweep curve is shown in 5.2(c). The origin translation vectors are shown.

In Figure 5.3(a) the swept area resulting from a single solid square at the origin swept through the curve from Figure 5.2(a) is shown. It can be interpreted as the impulse response or weighting pattern for the sweep. It represents the response for (result of) a solid square occuring at the origin or h(i,j) in (5-4).

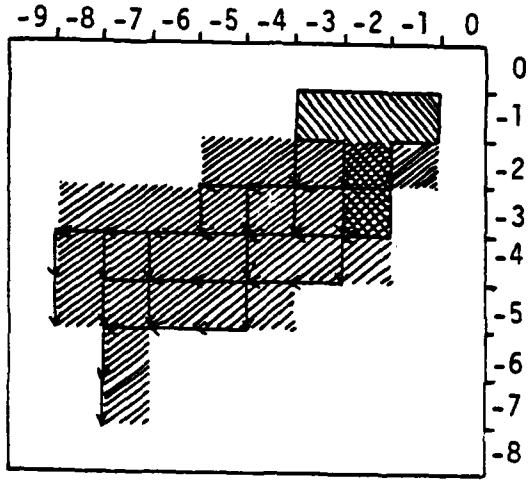
In Figure 5.3(b) the weighting pattern is reversed to form h(-i,-j). This is the pattern which will now be translated in the positive direction by k,m in (5-4) to form h(k-i,m-j). In Figure 5.3(c) the output array v(k,m) is shown. A particular square, v(1,2), is being evaluated. In 5.3(d), the weighting pattern is translated to place the origin over u(1,2). If solid squares for h and v are found for the same location, the value of v(1,2) is made solid. The AND operation at u(5,4) and h(-4,-2) is noted.



(a) Translation Curve

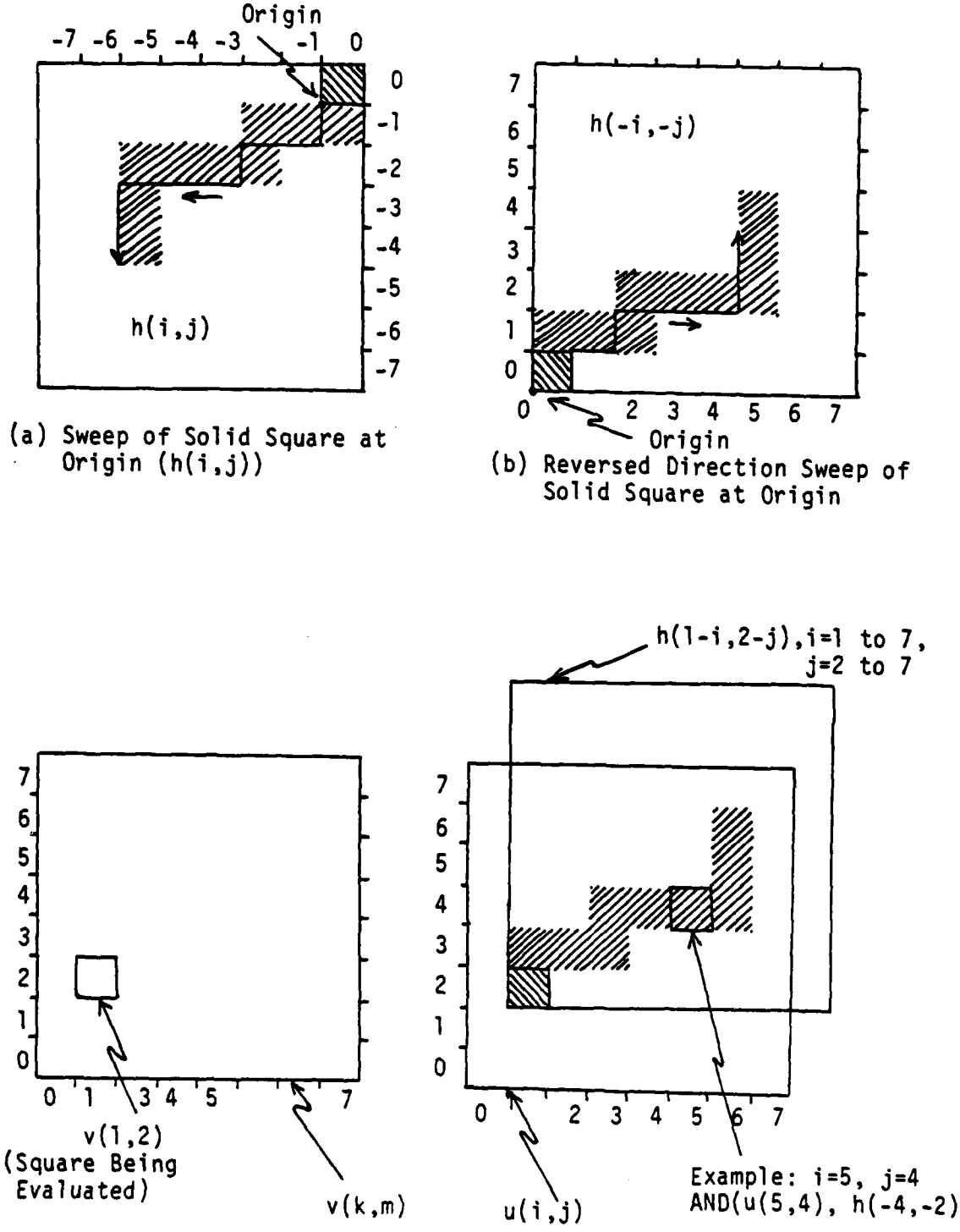


(b) T-Shaped Object



(c) Swept Area for T-Shaped Object

Figure 5.2 Swept Area



(a) Sweep of Solid Square at Origin (h(i,j))

(b) Reversed Direction Sweep of Solid Square at Origin

(c) Output Square Being Evaluated

(d) Location of Weighting Pattern When Evaluation v(1,2)

Figure 5.3 Weighting Pattern

5.2.3 Hierarchical Swept Volume

The above demonstrates that a translational swept area (or volume in 3-D) can be generated using a form of convolution but the technique cannot be directly applied to octree objects. In order to allow this, a "hierarchical convolution" summation was developed (see [12] for a similar technique applied to image processing). The weighting information is contained in an octree-like tree structure. The nodes contain weights for input obels just as the previous weighting function contained weights to be applied to locations in the input array.

Assume that an obel in an input octree object is being compared to an obel in a weighting tree (same level, same location). The value domain of the input obel contains P as well as the values empty (E) and solid (F) used above. As with previous octree algorithms, the general strategy will be to make a final decision if possible or, if not possible, defer to lower levels.

An "input/weighting node pair" or simply a "pair" is a set of two nodes at a common level, one from the input octree and one from a weighting tree. The input node represents a fixed obel in the input object's universe. The weighting node contains the weighting information to be applied to the input obel. Pairs defining a specific input/weighting evaluation are attached to nodes in the output tree being

generated. An output node may have many attached pairs awaiting evaluation.

If something solid contained within an input obel (an F node descendant) "causes" part of the output obel being evaluated to be solid (results in an F node descendant), it is said to contribute to the output obel.

The operation of the algorithm can be described in general terms as follows:

> For each output obel beginning at the root, maintain a list of input/weighting node pairs which could possibly contribute to it (the output obel). Starting at the root level, evaluate the pairs, eliminating non-contributing ones (those which evaluate to a value of E), passing appropriately ordered child pairs to the children of the output obel and their adjacent obel(s) (if not previously determined to be solid). Operation is terminated at the level of the chain code input.

Because of its hierarchical nature, the weighting tree cannot contain only binary values. Each node will contain one of four values with special meaning:

Value	Meaning
E	Nothing within corresponding input obel could contribute to output obel being evaluated.
P	Some solid part of input obel (F node descendant) could possibly contribute to output obel.
F	If input obel is not E (it's P or F) the output obel is solid (F).
P2	If input obel is F then output obel is solid (F). If input obel is P, some part of input obel could possibly contribute to output obel.

The weighting tree values are labeled in similar fashion to standard octree nodes for convenience and clarity. It is a special-purpose tree, however, and is only defined and meaningful within the context of this specific algorithm.

An input/weighting node pair is evaluated according to the following rules:

Rule No.	Input Node (u)	Weighting Node (h)	Evaluation (v=u/h)
1	E	(any)	E
2	(any)	E	E
3	P	P	P
4	F	P	P
5	P	F	F
6	F	F	F
7	P	P2	P
8	F	P2	F

The symbol "/" will be used to indicate an evaluation according to the above rules (pair evaluation = input node value/weighting node value).

For a particular output node, perhaps many pairs will be evaluated. If a pair evaluates to a value of E, the pair is simply deleted. If evaluated as F, the output obel is marked

F. If none evaluate to F, the output is P. Both F and P evaluated pairs are subdivided into multiple pairs for distribution to the children (if any) of the output node and the neighbors of the children (if not already F).

The child pair distribution pattern for a negative going sweep in 1-D is shown in Figure 5.4. Each node has two children. Each input/weighting node pair thus has four child pairs numbered 0 to 3. Two are passed down to child 0 of the node. One is passed to child 1 of the node and one is passed to child 1 of the neighbor node in the negative direction. Each child node thus receives about twice as many pairs as its parent, depending on pair elimination and the pair count of the parent's neighbor.

This pattern of child pair transfer is explained by considering the children of a pair. The two children of the input node will be u(i) and u(i+1). The two children of the weighting node will be h(k-i) and h(k-(i+1)). There are four combinations. Of particular interest is the combination of the earlier input with the later response value (child pair 1). It cannot contribute to the output children of the current output node. It forms one of the terms for the evaluation of an output node earlier in the sequence of output nodes (ie., smaller coordinate value than the output being evaluated by the original pair in the parent). The four pairs belong in evaluations as follows:

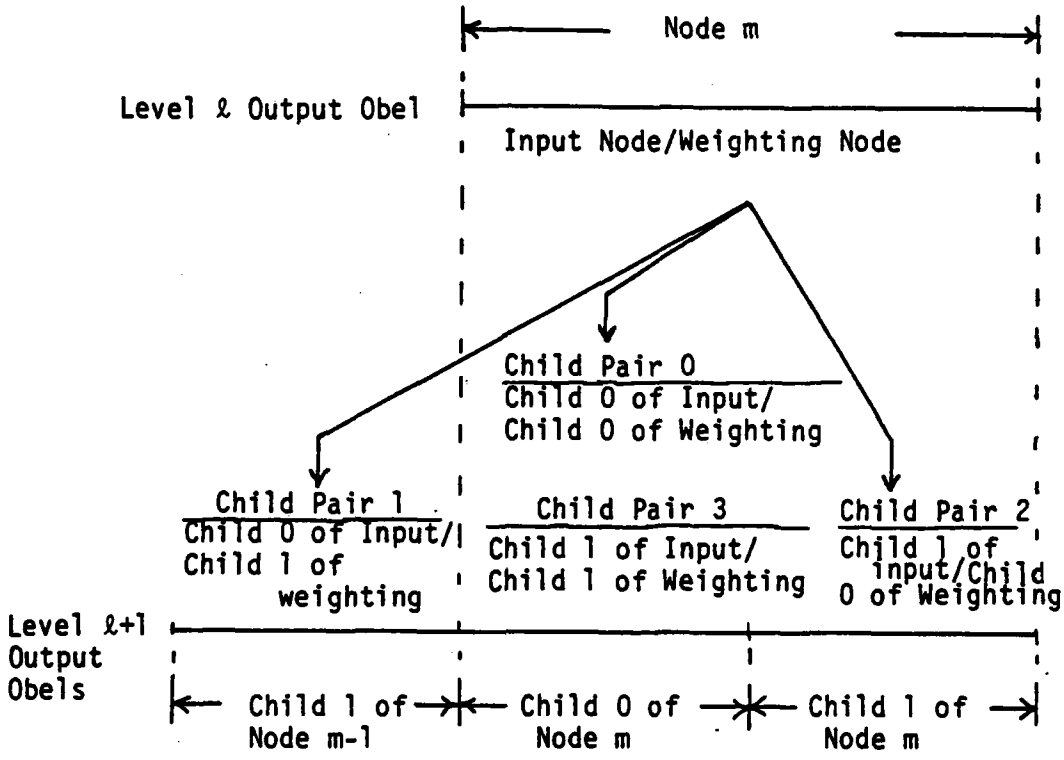


Figure 5.4 Distribution of Children Input Node/Weighting Node Pairs for 1-D Sweep Left

No.	Child Pair	Output Evaluation	Index
0	u(i)/h(k-i)	v(k)	i
1	u(i)/h(k-(i+1))	v(k-1)	i
2	u(i+1)/h(k-i)	v(k+1)	i+1
3	u(i+1)/h(k-(i+1))	v(k)	i+1

Note that output evaluations 0, 2 and 3 are for children of the original pair.

5.2.4 Weighting Tree Generation

The weighting tree is generated by sweeping test obels along the sweep curve. The process begins by placing at location 0,0 in an empty universe a single solid obel at level n, the level of the chain code represention of the curve. Conceptually, it is then swept using the standard translation and union algorithm along the curve but in the reverse direction (to account for the negative progression of the index for the weighting sequence). In practice, more efficient methods can be used.

A second copy is made of the sweep of the single test obel. This is the lowest level of the weighting tree. A third object is created by sweeping a test obel at level n-1 through the curve beginning, again, at the origin. Its nodes are then compared to the level n nodes in the original tree to determine the node values for the level n-1 nodes in the weighting tree. The following table lists the value generation rules:

Rule No.	Swept Test Obel -------- Status (current level)	Descendant Status (lower levels)	--- Output ---- Weighting Node (current level)
1	E	(any)	E
2	P	all E	E
3	P	not all E	P
4	F	all F	F
5	F	not all F	P2

The process is then repeated for level n-2 and above, up to the root at level 0. The resulting tree can then be reduced using the standard procedure. The P2 value is simply considered to be a P value.

The above rules can be analyzed by considering the situations individually. If a node is empty, clearly its descendants are also empty and nothing in the corresponding input obel can contribute. It is given the value E. For a P value, two cases exist. If the descendants are all E nodes, this higher-resolution information indicates that no contribution is possible and an E value is given. If they are not all E, the situation is ambiguous. The node is given a P value and the evaluation will be continued at the next lower level.

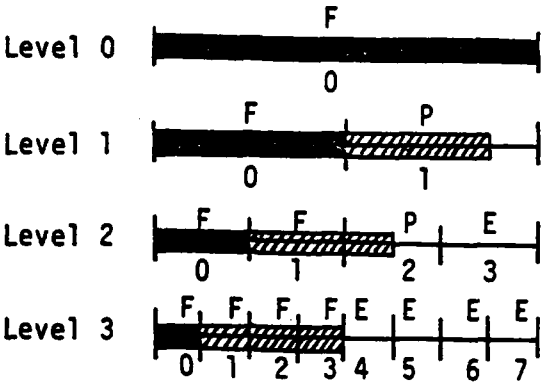
The fourth and fifth rules handle the two cases where the swept test obel has completely enclosed the weighting obel in question. If all the descendants are also F, the output obel will evaluate to F if the input obel is F or if any descendants (down to the level of the chain) is F (true if input tree is reduced and input node has value of P).

If not all the children are F, the information to be conveyed into the convolution operation is the following. If the corresponding input obel is solid (F) at this level, then the output obel being evaluated is also solid. If not, the result is ambiguous. The value P is given to the output and the final decision is deferred to the lower levels. The special value for such a weighting node is P2.

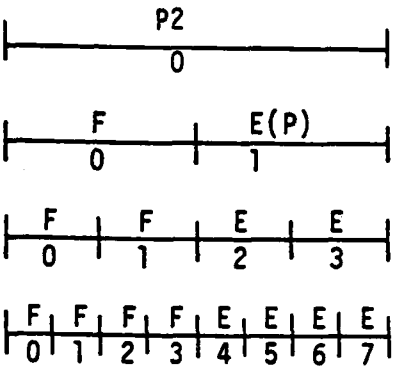
5.2.5 Example of Swept Volume (Hierarchical)

A 1-D sweep to the left will now be considered as an example. All objects will be swept left by three steps in a 4-level universe. First the weighting tree is generated. Figure 5.5(a) shows the result of sweeping a solid obel located at the origin to the right by 3 steps (reverse of curve direction) at the four levels. The test obel is solid and the added swept distance is shaded. At level 3, the solid obel is swept 3, resulting in the value F for all the children in the left half of the universe. At level 2, the swept distance extends into the right half. At level 1, the solid object is the left half of the universe. It is stepped by three level 3 units, covering all but one obel. At level 0, the test obel is the entire universe.

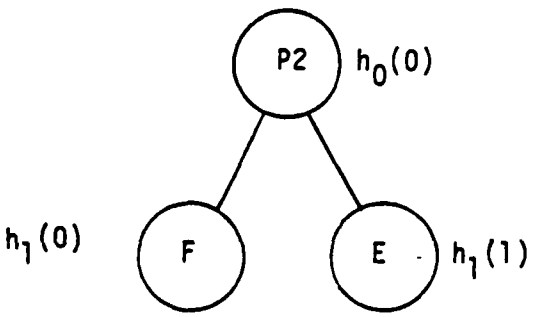
In Figure 5.5(b), the weighting tree is built up. At level 3, it is given the values from the sweep set at level 3. At level 2, the two left obels are marked F based on rule



(a) Four Sweep Right Level Sets



(b) Weighting Tree Values



(c) Weighting Tree h

Figure 5.5 Weighting Tree Generation

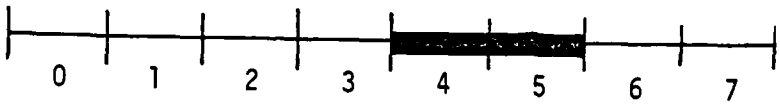
4. On the right, obel 2 is set to E based on rule 2 while 3 is set to E from rule 1. At level 1, node 0 becomes F from rule 4. Node 1 becomes P from rule 3. It is later reduced to a final value of E. The root is covered by rule 5, becoming P2.

Figure 5.5(c) is the resulting reduced weighting tree. Node labeling is as follows. Node $f_n(m)$ is the node from tree f at level n such that m is the decimal equivalent of its address string.

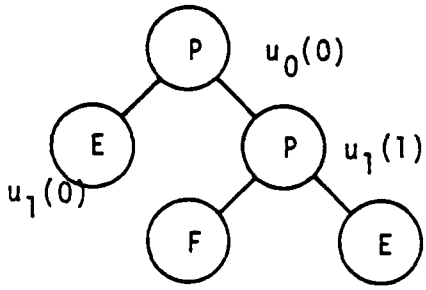
An object consisting of segments 4 and 5 as illustrated in Figure 5.6(a) will be swept. The object could, of course, be any 1-D object, including disjoint sections. The tree for the object is in Figure 5.6(b).

Figure 5.7 shows the steps as the algorithm proceeds. The resulting output object can be expanded to almost twice the size of the input universe. A second (empty) universe is added at level 0 on the negative side of the origin forming an augmented universe.

The positive universe at level 0 is initialized by attaching the pair containing the roots of the input and weighting trees (pair no. 1, P/P2). The pair is evaluated according to rule 7, resulting in the value P. It is thus continued into level 1 of the output universe according to the subdivision rules. The four sets (pairs 2 to 5) are noted at level 1 in Figure 5.7. All evaluate to E except pair 4 $(u_1(1)/h_1(0))$. It should be noted that even if all



(a) Object to be Swept



(b) Input (u) Tree of Object to be Swept

Figure 5.6 Object to be Swept

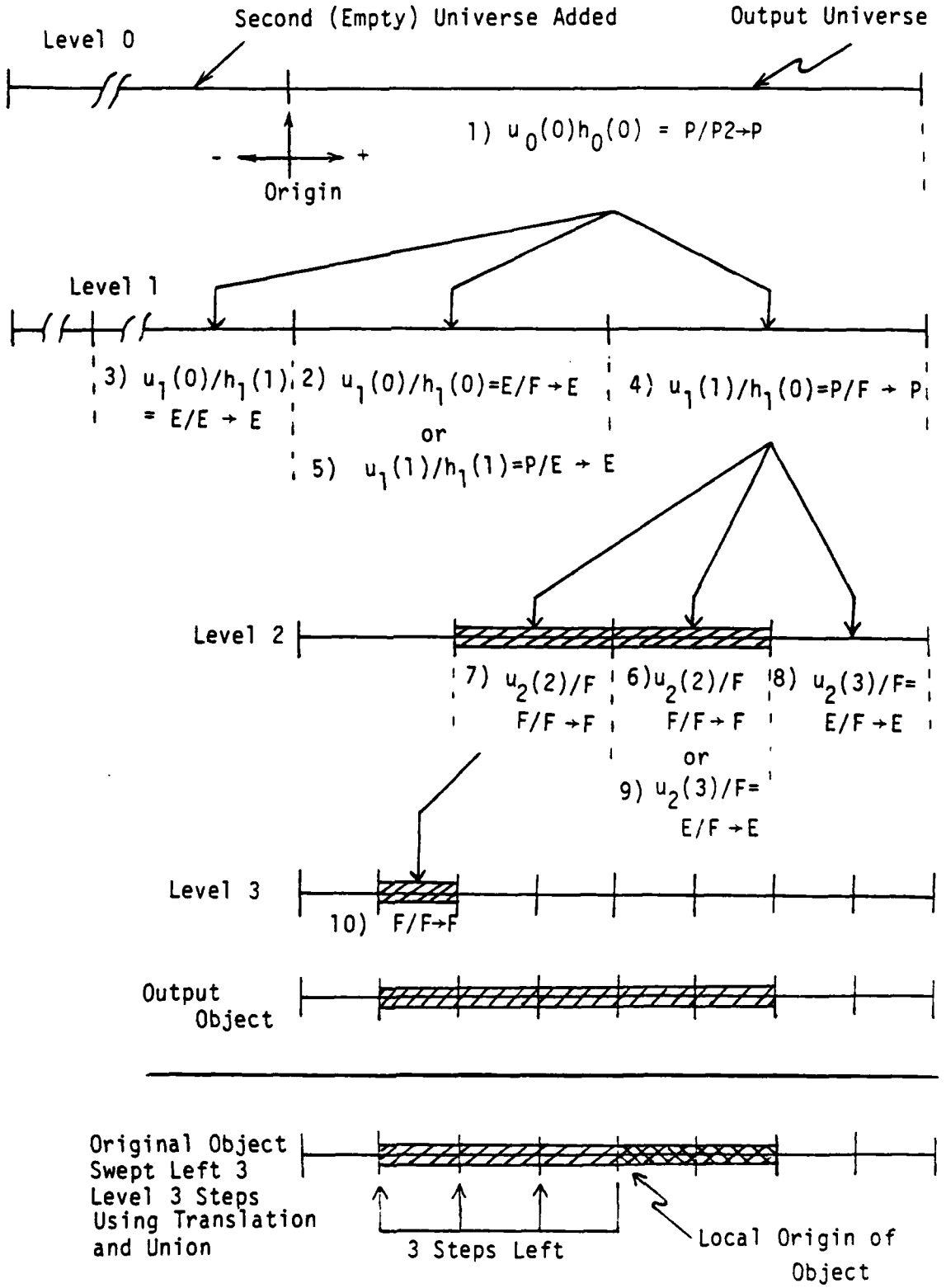


Figure 5.7 Example of 1-D Sweep Algorithm

pairs attached to an output node evaluate to E at this level, that output node will not necessarily be E in the final tree. An F value may be propagated in from a neighbor at a lower level. The output node is given a P value and will reduce to an E later if this fails to occur. A more sophisticated algorithm would check neighbor values first. If all are E, no F could be transferred in. A final value of E could safely be given.

At level 2, four sets are evaluated (pairs 6 to 9). The first of the two pairs in the center (pair 6) evaluates to F, determining a final output value of F for the node. The second is not considered until child pairs are being generated.

In a 1-D single direction sweep, only one neighbor of the children need be considered for child pair transfer. In this case (left sweep) it is the neighbor child to the left. For pair 7, an F/F evaluation is transfered to the left neighbor on level 3 (pair 10). Pair 6 has no child pair to propagate because it has evaluated to F and the neighbor to the left has already evaluated to F.

The original sweep chain was defined at level 3 and the process must be terminated here. Operation at lower levels will cause erroneous results as pairs are carried into neighbors.

The final output object is the new tree after reduction as noted in Figure 5.7. It consists of the two F nodes at

level 2 and the single F node at level 3. As a final check, the original object is swept left using conventional step and union operations, as shown at the bottom of the figure. The results are identical.

5.2.6 Analysis

For a generalized 1-D sweep in both directions, a weighting tree extended in the negative direction is required. The sets of pairs from the negative side would be generated as above but with a mirrored propagation pattern. In 2-D four sets of pairs are maintained and in 3-D, 8 sets.

One of the major applications of swept-volume techniques is in collision avoidance. In such a situation the desired end result is the spatial intersection of the swept volume with other objects in the universe. The actual swept-volume object is a temporary artifact generated in the process. Except in extreme situations, the vast bulk of the swept object is not involved in a collision and is, in a sense, wasted. If only the sections causing collision were calculated, perhaps substantial computational savings would result.

In the non-hierarchical convolution approach to swept volume represented by (5-4) this can be accomplished by simply evaluating the convolution only for locations within potential collision objects in the universe. There is no

need for evaluation to be performed on open space.

In the hierarchical approach, the situation is more complicated because pairs existing in open space could be transferred into a collision object at a lower level. It can be noted, however, that no descendants of a pair will ever extend beyond the space occupied by the immediate neighbors of the parent. Thus, for an output node in empty space (in the collision universe) if all its neighbors are also empty, all its associated pairs can be deleted. Otherwise, the child pairs are generated, but only for neighbors which are not empty (in the collision universe) and have not previously been determined to be completely occupied.

Most other octree operations involving two or more input objects, such as set operations, are linear in object complexity (a constant times the sum of the C values places an upper bound on calculations). They typically perform comparisons between objects over spatially identical sections of the universe. Simultaneous traversal over the input trees visits spatially identical obels because nodes with the same address represent identical sections of space. A single set of comparisons for each obel in the output tree is sufficient.

In the swept volume algorithm, however, the weighting tree does not represent absolute space. Weighting tree obels represent space relative to the location of the output obel being evaluated. Thus, it would be possible for every

weighting tree node to be examined for each input tree node (no pairs eliminated), in the worst case. Complexity is, therefore, on the order of the product of the C value for the input tree and the weighting tree. Because of pair elimination, much better performance could be expected in practice than this quadratic upper bound.

The swept-volume algorithm is easily extended for region growing and region shrinking. The "response" to a solid obel is an oversized or undersized solid obel. Such operations are useful for generating fillets, in trajectory planning, etc. Region shrinking can be used in a breadth-first traversal to determine the point of closest approach between two objects. The technique can also be modified slightly to perform cross correlation.

The swept volume algorithm can be viewed as an extension of the overlay concept. Instead of an output obel having a single overlay which determines its value, a series of overlays (list of pairs) is employed. The input/weighting node pairs specify not only the space in the immediate vicinity of the output obel but also spatially distant areas. A second extension has been the use of a special-purpose tree to specify the decision rules to be employed on a local basis. Previous algorithms could rely on a single universal decision set.

CHAPTER 6

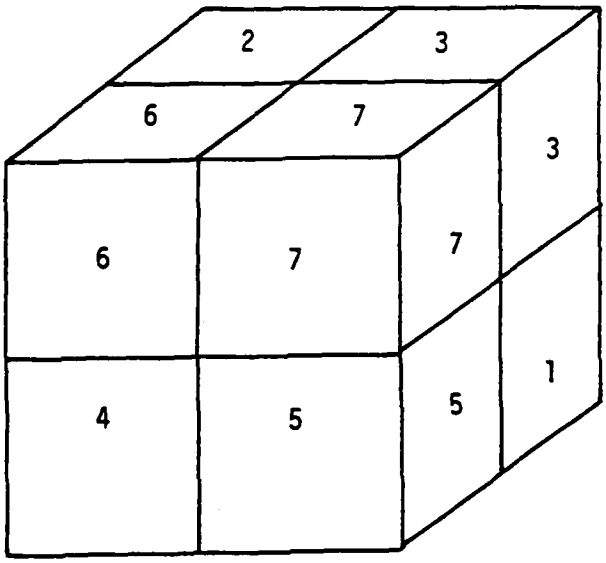
DISPLAY

The display algorithm accepts any number of input octrees and a set of view parameters (3 view angles, translation vector and a magnitude factor), and generates a quadtree containing an orthographic projection image of the objects, with shaded surfaces and hidden surfaces removed. Anti-aliasing is an integral part of the algorithm.

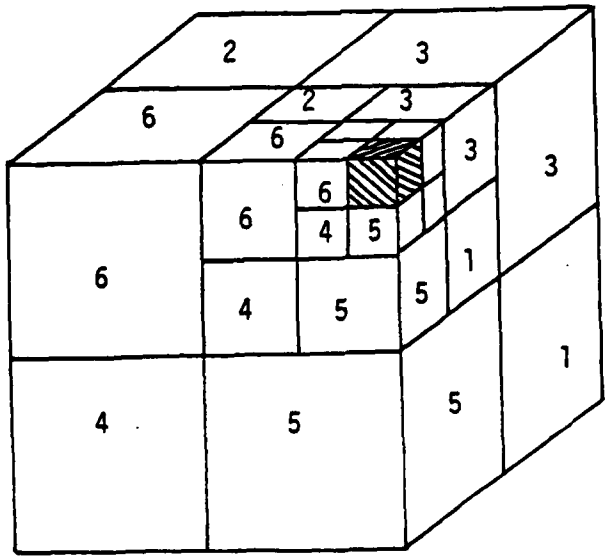
A second property, pixel intensity, is added to F (and possibly P) nodes in the output quadtree. The 2-D object then becomes a gray-scale image rather than a binary image.

The pre-sorted nature of the octree comes into play when generating a hidden surface view of one or more octree objects. By selecting the proper depth-first tree traversal sequence, the cubes corresponding to the visited nodes are accessed in a "back-to-front" or "front-to-back" sequence relative to the viewer.

This is demonstrated in Figure 6.1(a). From the viewpoint shown, nothing contained in cube 7 can be hidden from view by an object or part of an object in cubes 0 through 6. In like fashion, nothing in cube 6 can be hidden by anything in cubes 0 to 5, and so on. If the sequence 7 through 0 is followed recursively, a front-to-back sequence is generated. As shown in Figure 6.1(b) the first F cube is



(a) Hidden Surface Viewpoint for 7 to 0 Traversal Sequence



(b) Recursive Application of 7 to 0 Traversal Sequence

Figure 6.1 Hidden Surface Traversal Sequence

found by repeated selection of the highest numbered child.

Thus, a series of object parts is generated in which cubes in the sequence cannot be obscured by cubes later in the sequence. This eliminates the combinatorial explosion often encountered in hidden surface removal algorithms. No terminal node is accessed or examined more than once.

The new display algorithm presented here makes use of a front-to-back sequence. The visibility of each P node as well as each F node encountered in the sequence is checked against the parts of the image previously defined by earlier F nodes. If completely obscured, the node and all its children (if P) are discarded. Many hidden sections of the object or objects are not accessed and do not add to the computation. Thus the number of calculations required to generate an image is related to the visual complexity of the scene rather than the complexity of the objects.

Two tree structures are involved. One is a segmentation of object space (one or more input octrees) and the other, a segmentation of view space (the output quadtree).

The cubes represented by the nodes of the octree are projected on the display screen quadtree as shown in Figure 6.2. The projections consist of three 2-D four-sided polygons as shown in Figure 6.3(a). The bounding box of the polygon formed by the outer edges is also used as shown in Figure 6.3(b). In order to avoid confusion in the discussion to follow, the octree nodes and their projections will be

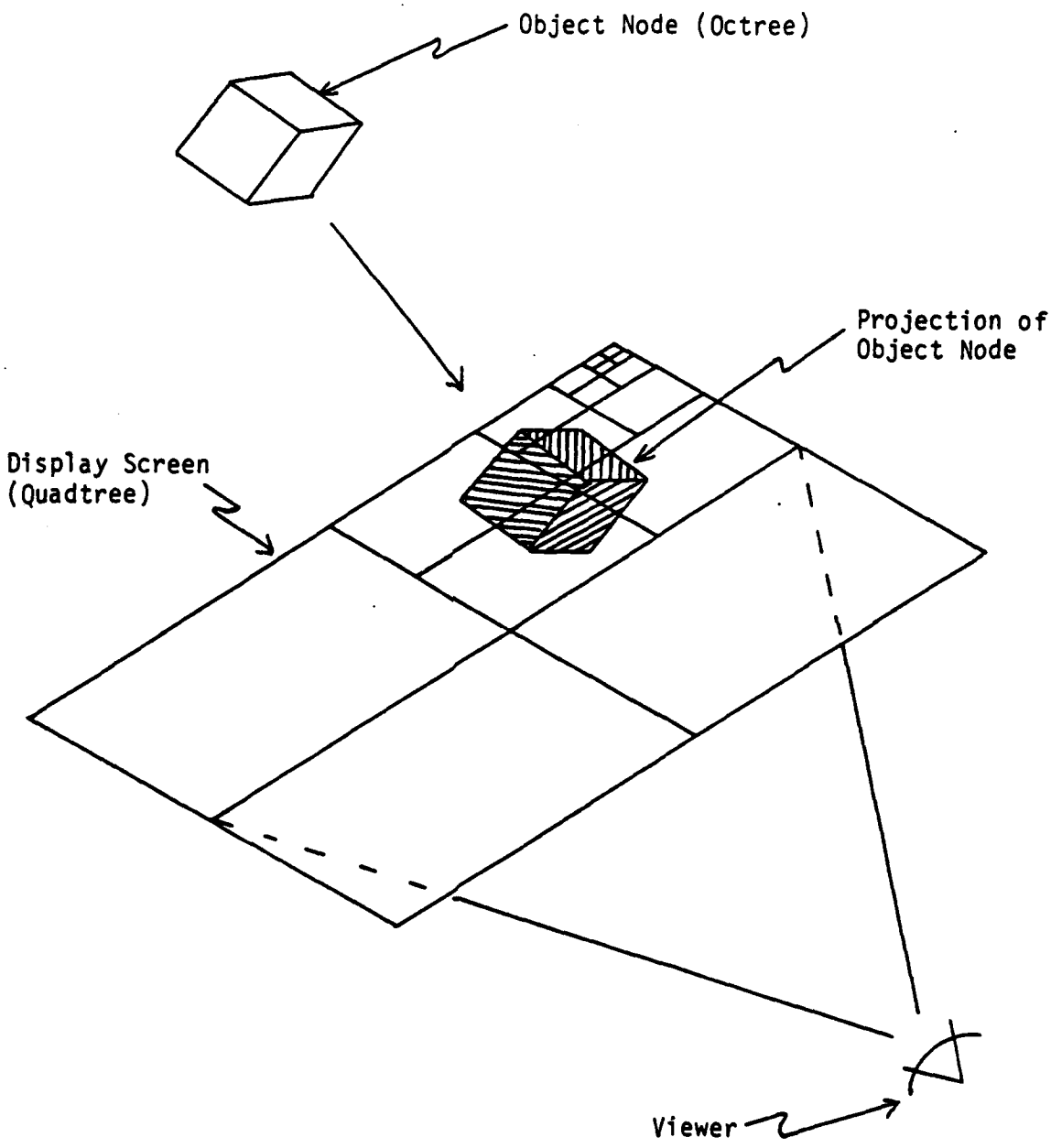
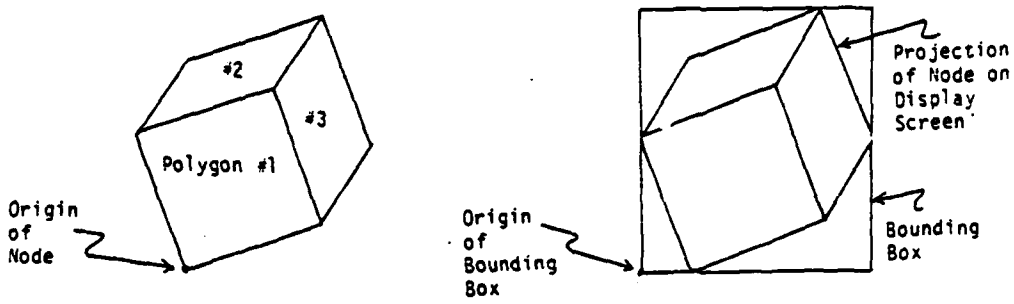


Figure 6.2 Octree Object Nodes (Cubes) are "Written" Into Display Screen Quadtree



(a) Projection of Node on Display Screen

(b) Bounding Box

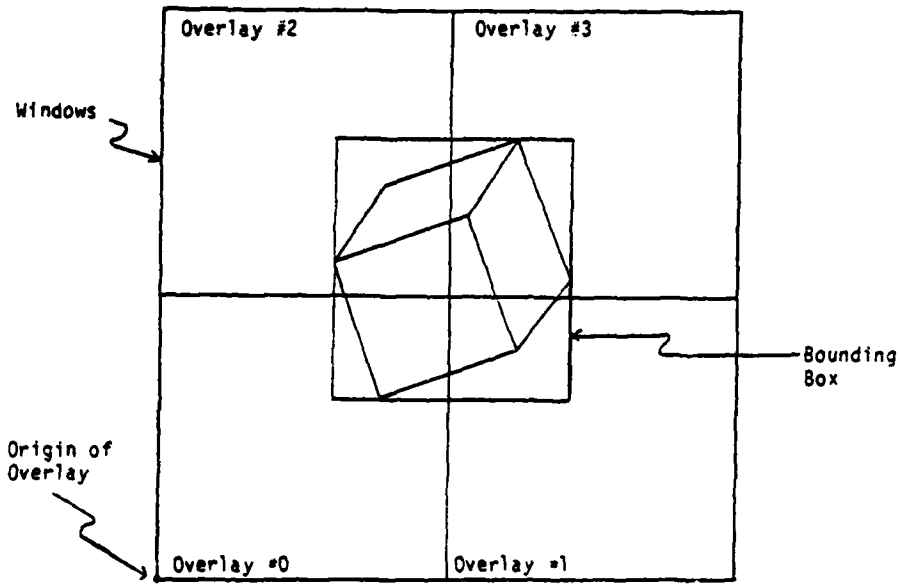


Figure 6.3 Projection of Node (Cube) on Display Screen and Overlay Structure

called simply "nodes." The nodes of the quadtree and the associated squares on the display screen will be called "windows."

The basic strategy is to access nodes in the front-to-back sequence and write them into the quadtree representing the display screen. Previously undefined windows enclosed by a node are given a corresponding intensity value and marked inactive. The quadtree is maintained in a reduced state during algorithm operation.

Each node accessed in the traversal (both branch and terminal) is checked for visual intersection with the quadtree. This can be performed using only simple arithmetic [43]. If it intersects only inactive windows, it is not visible and is discarded. The next node in the sequence at the same level or higher is then processed. If a branch node is eliminated in this way, all lower level descendants are automatically discarded without testing.

The computational load could still be unacceptable if the number of quadtree windows examined for each object node could become arbitrarily large. Once again, overlays will be used. The intersection test is performed between the node and exactly four overlay windows. The four windows are the four possibly intersecting squares at the lowest level such that the largest dimension of the bounding box of the node is the same size or smaller than the edge size of an overlay square. As shown in Figure 6.3(c), the origin of the

bounding box (lower left corner) is always in overlay number 0. It is clear that the four overlay squares will completely cover the object node.

Thus for each object node in the sequence tested, the intersection test is performed with four overlays. If all intersecting overlay windows are inactive, the node and anything within it are hidden and, therefore, ignored. If not, additional work is required. For branch nodes, the eight children are processed in like manner using the four covering overlay child windows of the parent overlay windows.

For terminal nodes, the children of the overlay windows are examined. Any active squares enclosed by the object node are loaded with the appropriate intensity value and marked inactive. At the minimum quadtree level, the center of the window is checked for enclosure. For a detailed description of the algorithm and the formal procedure, see [41].

For automatic sectioning, a blanking object is traversed in the same manner as displayed objects. All object nodes corresponding to an F node in the blanking object are converted to E nodes and therefore become invisible. Interference between objects can also be easily detected during algorithm operation.

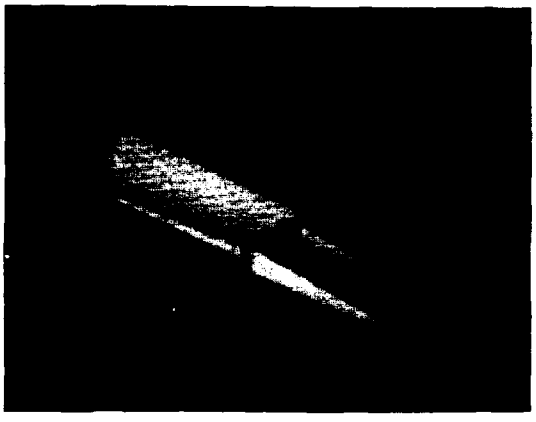
The intensity value attached to a quadtree node is the product of object color (or the texture value for that point on the object) and the cosine of the angle between the surface normal and the unit vector to the viewer (the dot

product of the two vectors).

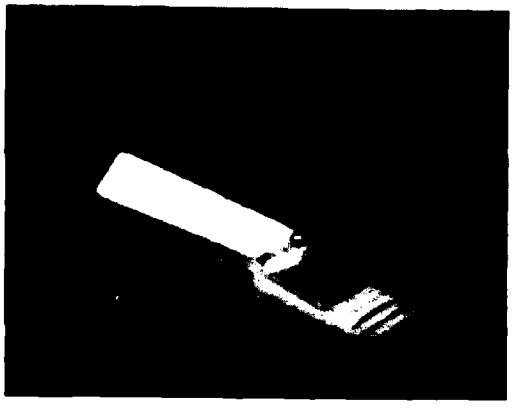
For objects defined mathematically, the surface normal is often easily calculated (or the surface normal times the texture value at that point) and the x, y and z components are attached to the node. This requires integer multiplication, may require substantial memory and, in some cases, the local surface normals are not easily computed. In such cases, block shading is used. The surface intensity values for the three faces of a cube are calculated. Quadtree windows enclosed by one of the three faces of a terminal cube are given the intensity value for that face. No multiplications are needed. This technique was used for the images presented in Chapter 7.

Figure 6.4 presents two images of a turbine blade represented in octree format. In 6.4(a), block shading was used. In 6.4(b) shading values were generated from unit normals attached to F nodes on the surface.

The anti-aliasing technique computes intensity values to a higher resolution than the screen. The averages of sets of these values are the pixel intensity values that are then displayed. The intersection and enclosure tests are performed to the lower levels. The intensity values for windows below the screen pixel level are summed into their ancestor windows at the pixel level. At display time, the pixel level window values are divided by the appropriate value (4, 16, ...) using shifts.



(a) Block Shading



(b) Surface - Normal Shading

Figure 6.4 Octree Representation of Turbine Blade

To add multiple illumination sources and shadows, the display algorithm is performed in reverse. A quadtree loaded with "illumination" is used. As shown in Figure 6.5, when nodes are projected on the quadtree they "pick up" and store whatever illumination is enclosed by the projection. The corresponding windows are then depleted and cannot illuminate additional nodes later in the sequence (they are marked inactive). This operation is performed once for each source. If an image is contained in the quadtree, it will be projected on the object or objects.

After illumination, the display algorithm is performed to generate an image. The accumulated illumination value for each node (or each face for block shading) is multiplied by the surface orientation weight (dot product of normal and unit vector to viewer) and, possibly, the texture value, to determine pixel intensity.

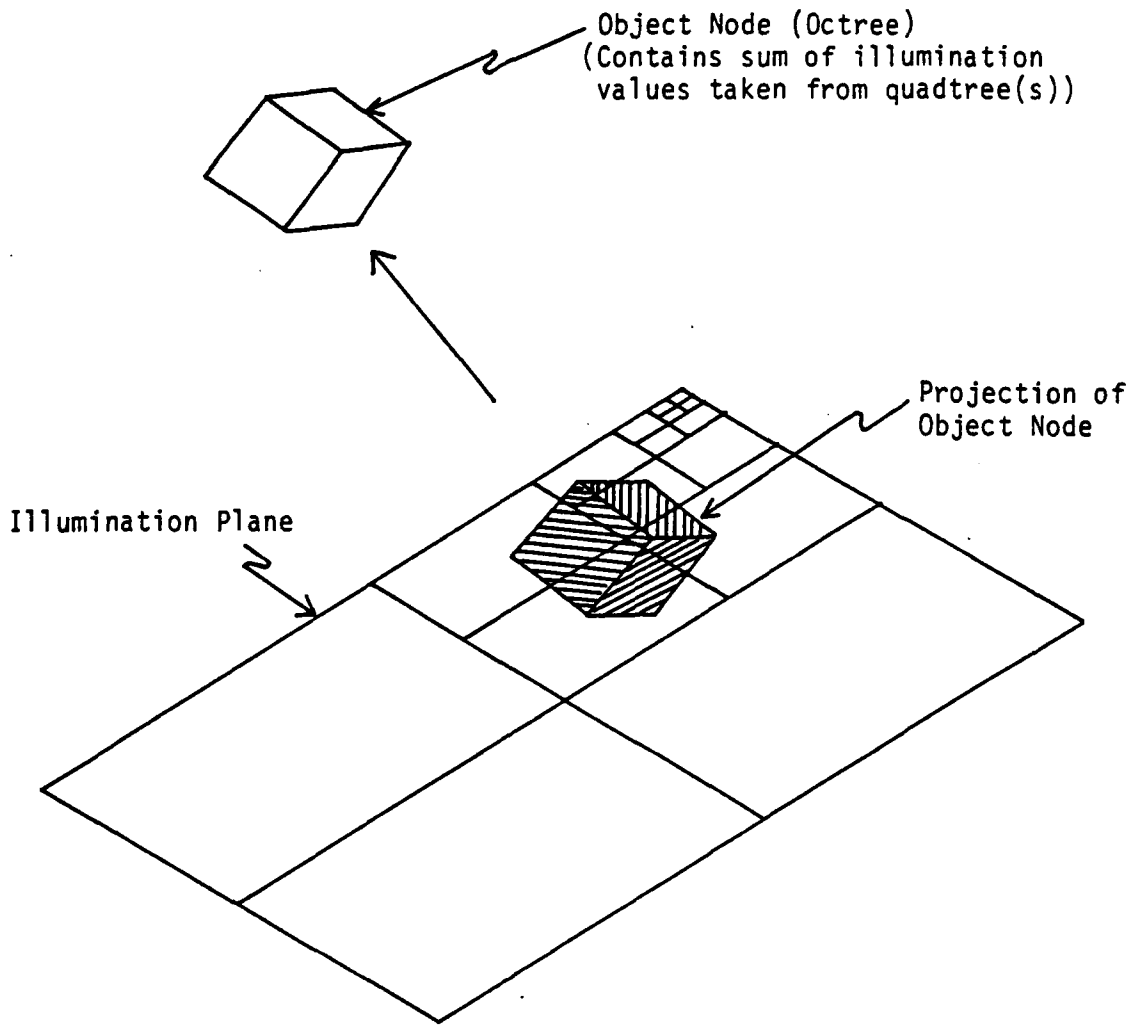


Figure 6.5 Illumination Plane (Quadtree) Projects Illumination on Object Node

CHAPTER 7

RESULTS

7.1 Program OCTREE

In order to develop, verify and demonstrate the octree encoding algorithms, the program OCTREE was written and implemented on the Prime 750 computer in the Image Processing Laboratory at RPI. A DeAnza IP5000 color imaging system was used for display.

The program currently encompases 257 routines containing 20,696 lines of Fortran code (44,864 if comments are included).

7.1.1 NC Verification

An important function of CAD is the generation of commands for NC (Numerical Control) machines [83]. A major problem has been the verification of control programs before release into production. Traditionally this has been performed on the machine tool itself and typically requires the program to be regenerated between three and five times

[26]. The verification task has become a bottleneck and a major expense in recent years as NC machines have become more popular, parts have become more complex and labor has become more expensive. Also, a substantial amount of capital equipment is removed from production and often tools (and sometimes the NC machine itself) are damaged.

There has recently been interest in utilizing solid modeling to replace current methods [84]. Ideally, a parts programmer or a machinist could verify all phases of an NC program interactively on a graphics CRT.

In the Fall of 1980, the Center for Manufacturing Productivity and Technology Transfer (CMP/TT) at RPI funded an effort through the author to develop a demonstration system to determine the feasibility of using octree encoding methods to perform NC verification.

An Imlac 6220 vector graphics terminal with lightpen was employed to facilitate user interaction. An algorithm was developed and implemented to extract the forward edges of an octree object for display.

In operation, the user specifies the workpiece (a block orthogonal to the axes) using the lightpen. Its octree is generated and displayed. A specific tool is then requested and loaded from a disc file. A top view of the workpiece is then presented. The user selects a starting location and an endpoint, again using the lightpen. A side view is next presented for entry of tool depth.

At this point the system generates an octree for the swept volume of the tool path and subtracts it from the workpiece. The tool and the workpiece are then displayed. The user can interactively change the viewpoint or request the next tool movement.

The program successfully demonstrated the viability of the technique and has continued to be developed and embellished by the CMP/TT staff.

Figure 7.1(a) shows a view of a simulated NC milling machine. The workpiece contains 40 nodes, the clamp 200 nodes, and the tool 7432 nodes for the bit, 40 nodes for the shaft and 40 nodes for the holder. All 5 objects together require less than 2000 bytes of storage (serial allocation at 2 bits/node).

In Figure 7.1(b) a channel has been machined in the workpiece. Figures 7.1 (c) through (f) show the top, front, side and orthogonal views after additional material has been removed.

Using currently available algorithms the program could be given the capability to detect and display any part of the original workpiece outside the maximum desired object, any part of the minimum object removed, and the tolerance object still remaining after the machining operation.

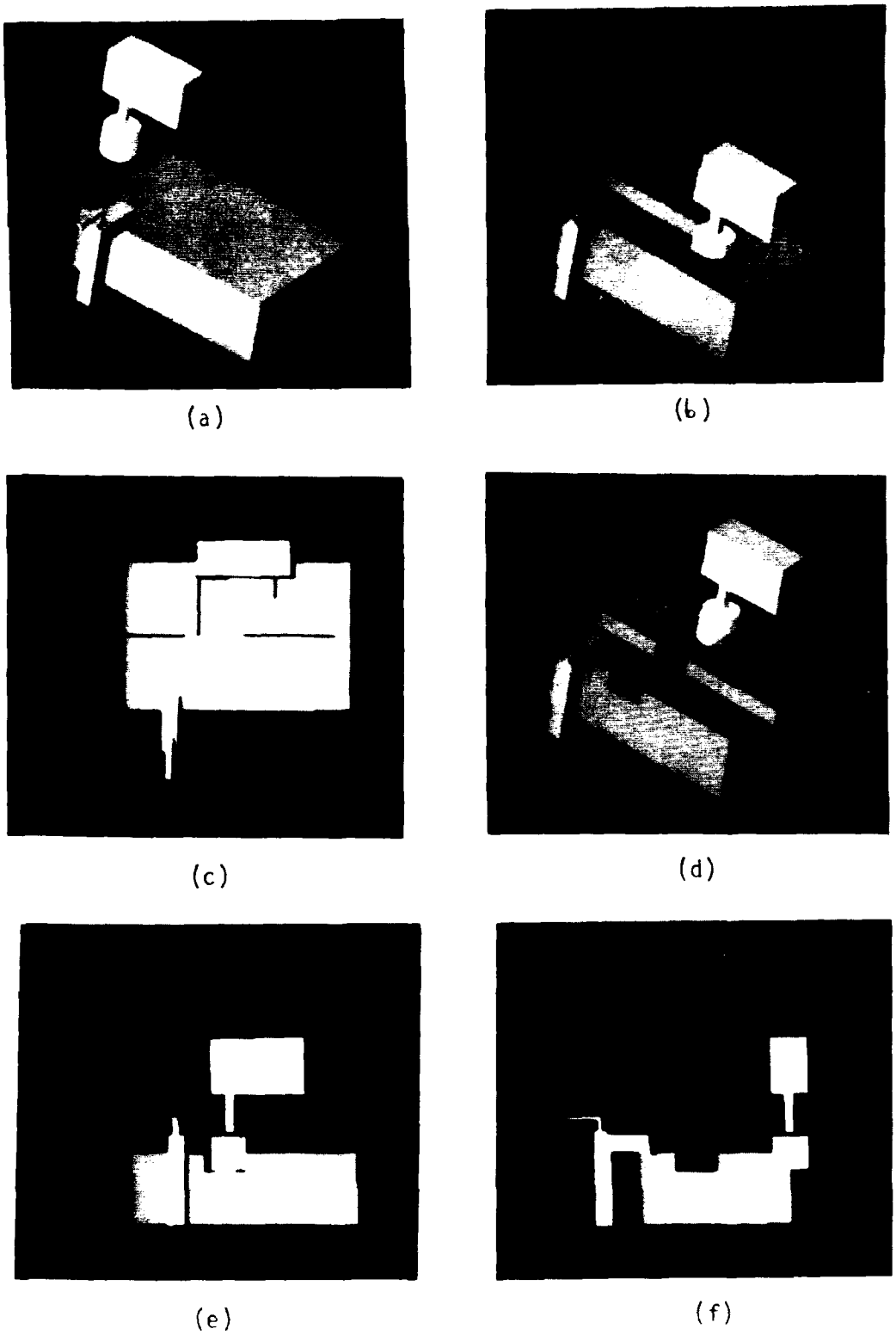


Figure 7.1 Simulated Milling Operation

7.1.2 Medical Imaging

In the spring and summer of 1981, an effort funded by Phoenix Data Systems, Albany, New York, was undertaken by the author to investigate the possibility of using octree encoding methods to interactively generate medical images derived from CT scans [2, 73, 75-77]. Such a capability would be useful in pre-operative surgical planning [2] and in other areas. This section reviews the preliminary results.

A series of 42 CT scans of a human head were used for demonstration and evaluation (scans courtesy Dr. Gabor Herman, formerly Director of the Medical Image Processing Group, State University of New York at Buffalo). Thresholded binary octree objects were generated for display [86]. Figure 7.2 presents 4 views of the section of the skull contained in the images. The section extends from just below to just above eye level. Images 7.2(a) and 7.2(b) are sectioned views of 7.2(c) and 7.2(d), respectively.

The skull octree in 7.2(c) and 7.2(d) requires 43,112 bytes for storage using serial allocation at 2 bits/node (C=172,449) whereas the sectioned skull object in 7.2(a) requires 22,234 bytes (C=88,937). The original set of images (with CT numbers) required over 5.5 million bytes for storage. A more valid compression comparison would be with the set of thresholded binary images. This would correspond

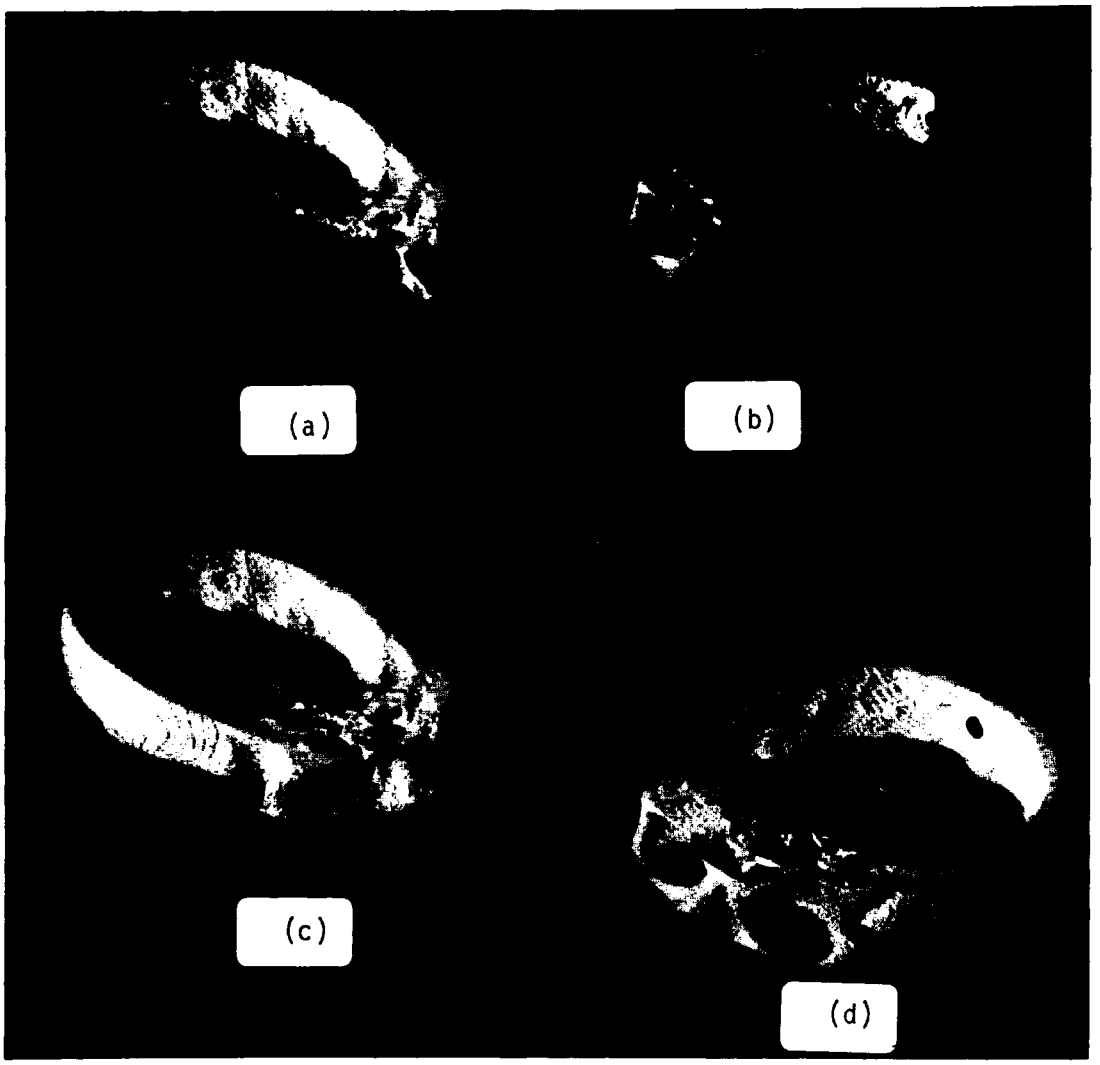


Figure 7.2 Display of Section of Human Skull

to a spatial enumeration representation. In compressed format (1 bit/voxel) the object would require 344,064 bytes or approximately 8 times the storage required for octree storage of this object.

Figure 7.3(a) is an "object" which represents the low density areas inside the head. It corresponds to the sinus passages. The forward section forming the nasal passages has been removed to improve visibility.

In order to understand its location, in Figure 7.3(b) the sinus object has been sectioned and embedded in the back half of the skull from Figure 7.2(b).

A different density thresholding scheme was used to generate the sinus object used in Figures 7.3(c) and 7.3(d). A narrow threshold corresponding to the air-tissue interface was selected. Image 7.3(c) is a sectioned view similar to 7.3(b). The removal of low density air accounts for the "hollowed out" look which can be noted in 7.3(c).

The patch forward of the eye socket is composed of part of the eyelid and the beginnings of the skin on the nose (the forward part of the nose is not contained in the CT images).

Figure 7.3(d) is a forward looking view of the entire sinus object of 7.3(c) plus the opposite section of the skull. The thin, almost vertical part on the right of 7.3(d) is the back side of the patch of skin in 7.3(c).

Figure 7.2(d) was generated using the display algorithm presented above. Almost 400,000 level transitions were

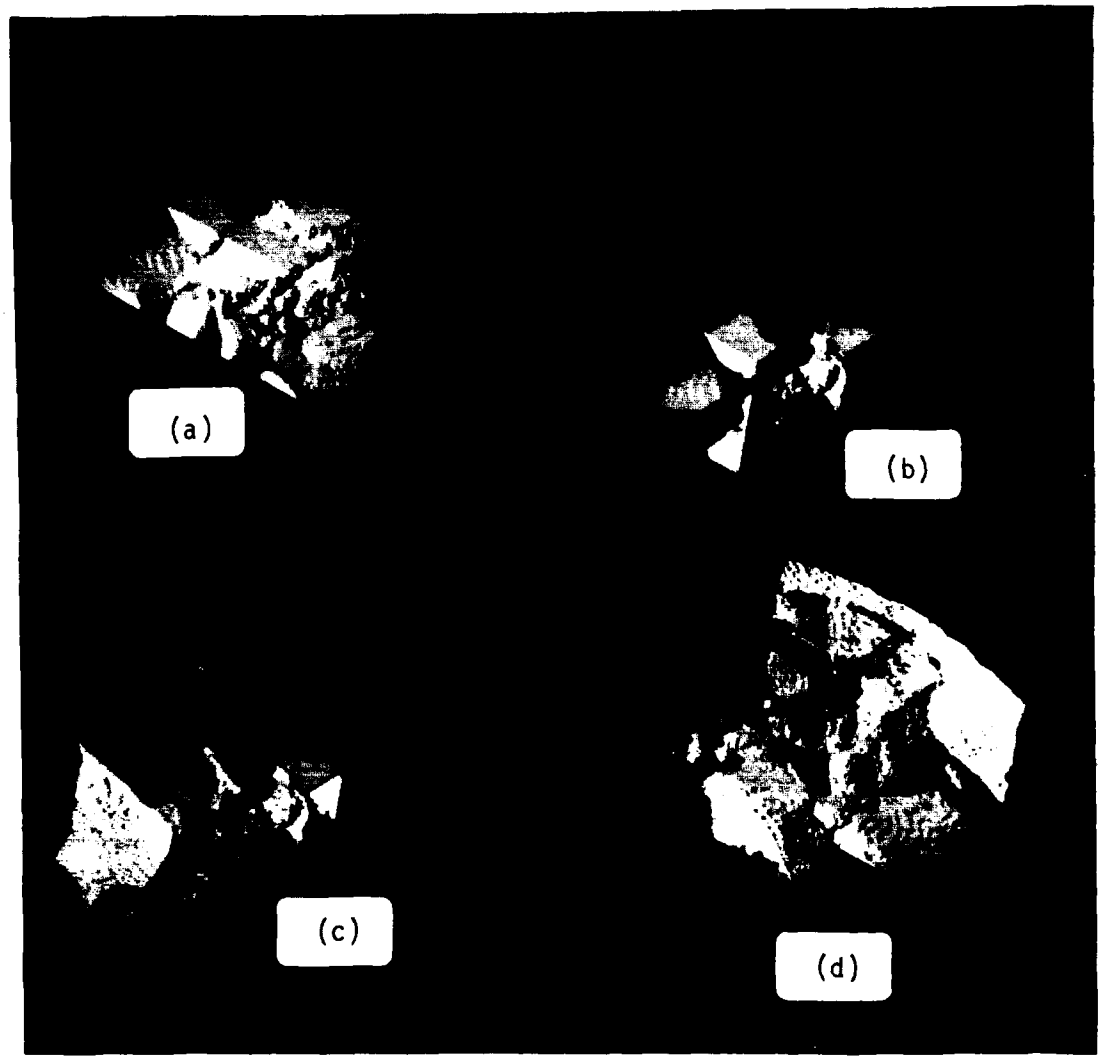


Figure 7.3 Display of Sinus Passages

needed, requiring about 11 minutes of CPU time on a Prime 750. Approximately 60% of the octree nodes were accessed.

An earlier back-to-front display algorithm [39] was used to create the remaining images in Figures 7.1 through 7.3.

An algorithm was also developed to display transparent objects using only simple arithmetic. The resulting images are very similar to those generated using conventional transmission x-ray techniques.

CHAPTER 8

DISCUSSION AND CONCLUSIONS

The viability of octree encoding as a solid modeling method has been demonstrated in two significant examples, NC verification and medical imaging.

At the current state of development, efficient algorithms exist to perform the following:

Octree generation from: convex polyhedra, restricted convex shapes defined mathematically, edges of planar sections (in chain code format), multiple 2-D images, intersecting prisms, spatial enumeration arrays, lists of surface intercept points and swept 2-D or 3-D objects.

Object property measurement: volume, surface area, center of mass, moment of inertia, number of interior voids, correlation, and closest approach between 2 objects.

Object operations: union, intersection, difference, negation, segmentation of disjoint parts, space filling of bounded volume, translation, scaling (power of 2 or arbitrary factor), rotation (multiple of 90 degrees or arbitrary angle), skewing, reflection, linear and nonlinear transformations, region growing and shrinking, sectioning, perspective deformation, and the generation of translational or rotational swept volumes.

Interference detection: for fixed or swept objects.

Display: raster image generation from any viewpoint with hidden surface removal, orthographic or perspective view, black-and-white or color, surface normal or block shading, shadowing, dynamic movement of light source, multiple illumination sources, automatic anti-aliasing, sectioning, interference detection, transparent objects, textured surfaces, back-to-front or front-to-back traversal, and raster or quadtree output. Also, vector display of forward edges of objects (generation and display of edge-tagged objects).

The octree method is entirely new and not directly rooted in any existing system. It is difficult directly to compare this technique with other SMS methods. Rather, a profile will be obtained by analyzing its characteristics with respect to the 21 problem areas presented in Section 1.1.

(1) Limited domain - The domain of objects is completely unlimited within the restrictions of precision and storage imposed by a specific implementation.

(2) Validity - Any legally generated octree is valid.

(3) Completeness - All octrees are complete. No overhead processing is required to assure this.

(4) Uniqueness - A reduced octree is unique if fixed in location and orientation. The octree method thus has permutational uniqueness but not positional uniqueness.

(5) Conciseness - In general, conciseness ranges from

poor for standard primitive shapes to very favorable for more arbitrary objects (eg., skulls). One advantage is that, in its reduced form, there is no redundant information. No section of space is represented twice.

(6) Closure - The octree encoding method is closed.

(7) Finiteness - All octree objects are finite.

(8) Null object - The null octree is simply an E root node (in reduced form).

(9) Transportability - Results should be identical from one system to another because of the strictly integer arithmetic. This assumes identical algorithms, scale factors, etc. It is possible that, unless care is taken, implementation-specific differences could cause changes in the conversion of user requests to the integer format required by the algorithms.

(10) - Extensibility - In general, octree encoding is easily extended with respect to object size, the number of objects, object complexity, precision, etc. Assuming that the dynamic range of integer variables is not exceeded, standard octree processors should be extensible in these areas almost without limit.

(11) Autonomy - The algorithms developed to date are autonomous.

(12) Reliability - All objects which could possibly be generated can be processed by all compatible algorithms. A higher level of intelligence is not needed for guidance to

handle unusual or special cases.

(13) Efficiency - Within the definition of object complexity (the value of C), the basic manipulation, analysis and display algorithms are linear in the number and complexity of objects. In practice, many algorithms such as display and interference detection, do not require all object nodes to be processed.

(14) Implementability - Octree Encoding algorithms should be easily implemented in multiple VLSI processors operating in parallel.

(15) Multiple representations - In operation, two data structures are envisioned, an application data base and an octree structure used for interactive processing.

(16) Consistency - Consistency is not expected to be a problem because, in most cases, object conversion will be required only from application formats into octrees.

(17) Conversion - Conversion to octree format is a straightforward process for most current representation schemes.

(18) Ease of object creation and manipulation - Given high-speed conversion capability and interactive octree processors, performance in this area should be high when compared to other methods.

(19) Finite-element modeling capability - The use of quadtree and octree techniques to automatically generate FEM meshes was suggested (but not developed) by the author. This

area is currently under active investigation at RPI [87]. Preliminary results have been very promising. The automatic placement of smaller squares and cubes in areas of high curvature has been found to be a useful characteristic.

(20) Interference analysis - Interference detection is performed in linear time. In many cases it is related to situation complexity. This is a large improvement over most competing techniques.

(21) Tweaking - Local modifications can be performed in isolation because of the spatially sorted nature of the octree.

8.1 Accomplishments

A solid modeling method based on a hierarchical tree structure, the octree, was designed and developed. It is believed this new method may allow, for the first time, the construction of relatively inexpensive full-function solid modeling systems capable of handling large numbers of complex objects at real-time or near real-time rates. This includes interference related operations such as interference detection, collision avoidance and hidden surface removal.

The following are considered to be the most significant specific results:

(1) Development of the octree methodology for solid

modeling, with emphasis on performing operations efficiently using simple integer arithmetic.

(2) An efficient display algorithm based on the use of front-to-back octree traversal for hidden surface removal.

(3) A swept-volume generation (and collision avoidance) algorithm based on hierarchical convolution.

(4) An efficient interference detection algorithm based on octrees.

8.2 Suggestions for Further Research

The following areas are suggested for further research:

(1) Alternate object representation format - A possible research area could be the investigation and analysis of alternate formats. For example, obels could be subdivided into 27 children rather than 8 or subdivided in only one dimension at each level. This might lead to more compact representations.

Also, octree methods could probably be used by other solid modeling schemes to improve efficiency.

(2) Object generation - Efficient conversion methods are needed for object representations such as ruled surfaces, surface patches and various parametric formats. Automatic octree generation from wireframe

projections [37, 85] could be useful.

(3) Additional operations - Additional analysis and manipulation techniques would be useful, such as calculation of curvature and surface normals, volume conserving deformations, generation of blends and extrusions, extracting edges for drafting applications, automatic dimensioning, etc.

(4) Shape classification - It would be desirable to develop some form of automatic classification of octrees by shape. Perhaps a start would be to decompose octrees into primitive shapes or edge/face/vertex sets with connectivity information.

(5) Advanced finite-element modeling - It may be possible to utilize octree techniques in finite-element analysis.

CHAPTER 9

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Solid modeling is the subject of representing solid objects in a computer - to permit their analysis, manipulation and display. This thesis describes the development of a new solid modeling method called octree encoding, in which arbitrary objects are represented to a specified resolution in 8-ary hierarchical trees or "octrees." The number of nodes in an object's octree is used as a measure of object complexity. This number is shown to be on the order of the product of object surface area and inverse of the square of the resolution. (over)

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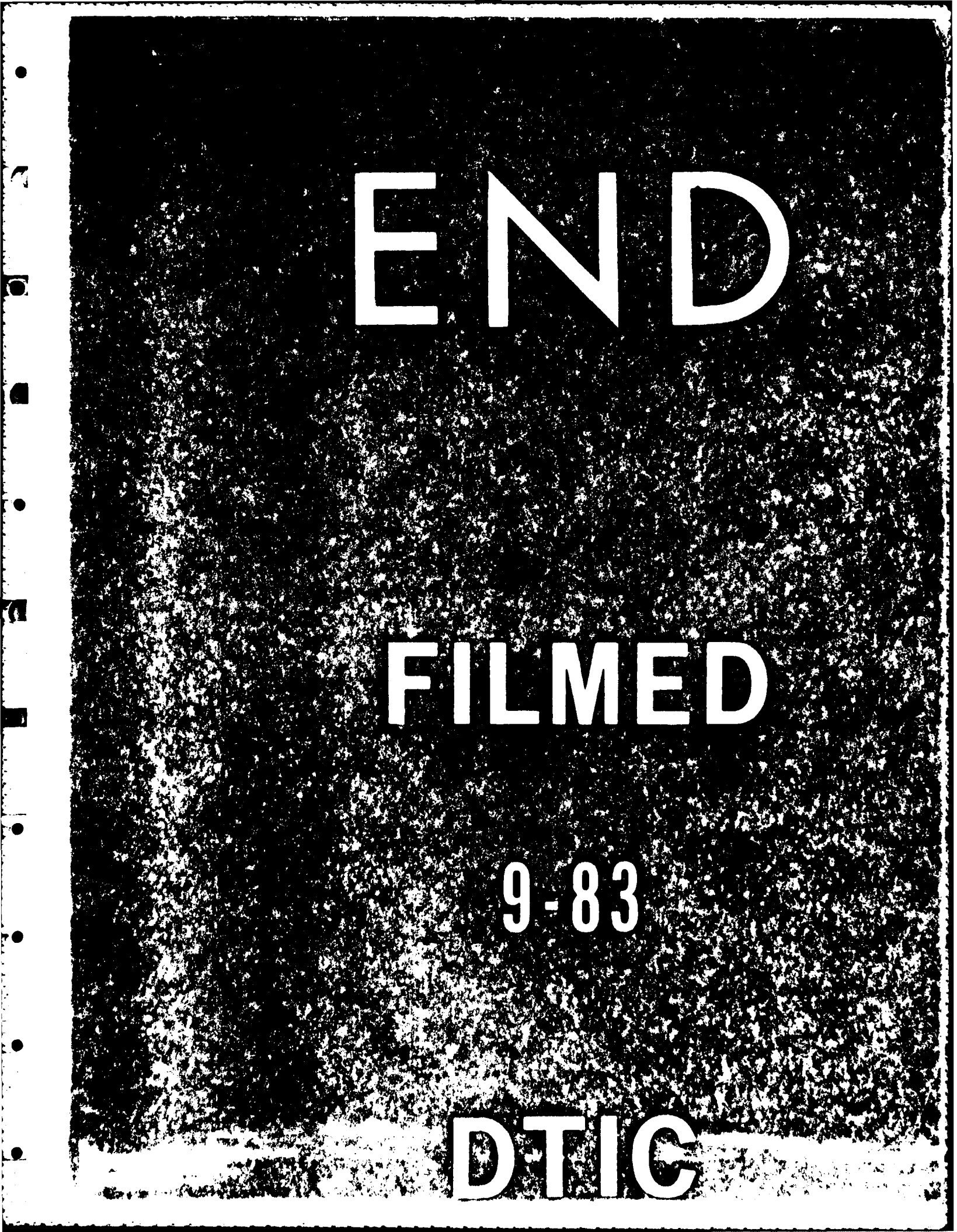
A dual data-base approach is proposed. A general-purpose solid-modeling system based on octree encoding would interactively perform geometric, analytical and display operations in conjunction with specialized application data bases.

Efficient algorithms are presented for the determination of mass properties (volume, surface area, center of mass and moment of inertia, etc.) for the formation of new objects via the use of set operations (union, intersection, difference and negation), for linear transformations (including translation, scaling and rotation), for interference detection, for swept-volume definition, and for display from any point in space (with surface texture, anti-aliasing and hidden surface removal). The complexity of the processing required to display an object is related to the visual complexity of the scene rather than the complexity of all objects involved. Interference detection requires computation related to the separation distance between the objects.

The above algorithms require only simple integer arithmetic (addition, subtraction, magnitude comparison, and shift) in order to facilitate implemention in VLSI processors.

The new method is compared to existing solid modeling methods in 21 problem areas.

Results are presented which show the application of the technique in the verification of NC (Numerical Control) machine programming and in the display of 3-D medical objects derived from multiple CT (Computed Tomography) images.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